

**Rishi Bankim Chandra Evening College**  
**B.Sc. (General) 4<sup>th</sup> Semester Internal Examination**  
**Subject – Algebra**  
**Subject Code – MTMGCOR04T**

**Time – 60 minutes**

**F.M.-20**

**Answer any ten questions from the following**

**10×2=20**

1. Prove that  $(A - B) \cap B = \emptyset$  is true for any two sets A and B.
2. If  $S = \{-3, -2, -1, 0, 1, 2, 3\}$ , is  $(S, +)$  a group?
3. Let the function  $f: R \rightarrow R$  be defined by  $f(x) = 2x + 3, x \in R$ , If  $A = \{x: 1 \leq x \leq 2\}$ , find  $f(A)$ .
4. In a group G, prove that  $\forall a, b \in G, (ab)^{-1} = b^{-1}a^{-1}$ .
5. In a group G, if  $\forall a \in G, a^2 = e$ , then show that G is commutative group.
6. Define field.
7. Assuming that the set, Z of all integers form a commutative ring under addition and multiplication, show that Z is not a field.
8. Show that intersection of two subgroups of a group is also a subgroup of that group.
9. Show by an example that the union of two subgroups of a group may not be a subgroup of that group.
10. If  $A = \{1, 2\}$ ,  $B = \{1, 2, 3\}$ , find  $(A \times B) \cap (B \times A)$ .
11. Show that mapping  $f: R \rightarrow R$  given by  $f(x) = 2x + 3$  is bijective.
12. If two mappings  $f: R \rightarrow R$  and  $g: R \rightarrow R$  are defined as follows  $f: x \rightarrow x^2, g: x \rightarrow x - 2$ , then show that  $f \circ g \neq g \circ f$ .
13. Let  $G = \{1, \omega, \omega^2\}$  be the set of three cube roots of unity. Show that G is a group under the usual multiplication of complex numbers.
14. Show that a group  $(G, *)$  is commutative iff  $(a * b)^2 = a^2 * b^2, \forall a, b \in G$ .
15. Prove that the set of all even integers, under the usual addition, is a subgroup of the group of integers.
16. Let R be a ring with unity element 1. Show that  $(-1)(-1)=1$ .