



WEST BENGAL STATE UNIVERSITY
B.Sc. Programme 6th Semester Examination, 2021

MTMGDSE03T-MATHEMATICS (DSE2)
NUMERICAL METHODS

Time Allotted: 2 Hours

Full Marks: 50

*The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.*

Answer Question No. 1 and any five from the rest

1. Answer any *five* questions from the following: 2×5 = 10
- (a) Construct a linear interpolation for $f(x)$ with $f(1) = 3$ and $f(2) = -5$.
- (b) Compute $\int_0^4 2x^3 dx$, by Simpson's $\frac{1}{3}$ formula and comment on the result.
- (c) Derive an iteration formula for computing $\sqrt[3]{a}$, using Newton Raphson method.
- (d) What is the condition of convergency of Gauss-Jacobi iteration to solve the system of n linear equations? Is this condition both necessary and sufficient?
- (e) Show that the equation $x^2 + \ln x = 0$ has exactly one root in the interval $\left[\frac{1}{3}, 1\right]$.
- (f) If 0.667 be an approximate value of $\frac{2}{3}$, find the percentage error.
- (g) What do you mean by Numerical Differentiation?
- (h) Show that $\Delta^2 \cos 2x = 4 \cos 2x$ where interval of differencing is $\frac{\pi}{2}$.
- (i) Define the terms absolute and relative errors.
2. Explain the Newton-Raphson method for computing a simple real root of an equation $f(x) = 0$. When does the method fail? Can we apply this method to the equation $x^2 - x + \frac{1}{4} = 0$? Justify your answer. 4+1+1+2
3. (a) In order to find the root of $x^3 - x - 1 = 0$, near $x = 1$ which of the following iteration functions give convergent sequences: 2+2
- (i) $x = \frac{x+1}{x^2}$ (ii) $x = \sqrt{\frac{x+1}{x}}$
- (b) Apply the method of bisection to find a real root up to two significant digits of the equation $x^3 - 3x - 5 = 0$. 4

4. (a) Use Lagrange's interpolation to find the value of $f(x)$ for $x = 0.4$ using the table. 5

x	0.3	0.5	0.6
$f(x)$	0.61	0.69	0.72

- (b) Find $\Delta^4 f(x)$, where $f(x) = (3x+2)(x-2)(x+1)(5x-1)$ and the interval of differencing is unity. 3

5. What is interpolation? Deduce Newton's forward difference interpolation formula without error term. 8

6. (a) Given the following table: 2+3

x	0	5	10	15	20
$f(x)$	1.0	1.6	3.8	8.2	15.4

Construct the difference table and compute $f'(20)$.

- (b) If $f(x) = ax$, show that $(E + E^{-1})f(x) = 2f(x)$. 3

7. (a) Calculate $\int_1^2 (x + \frac{1}{x}) dx$ up to four significant figures by Simpson's $\frac{1}{3}$ rule taking 4 intervals. 4

- (b) Obtain trapezoidal rule for numerical integration without the error term. 4

8. Solve the system of equations by LU decomposition method: 8

$$3x + 4y + 2z = 15, \quad 5x + 2y + z = 18, \quad 2x + 3y + 2z = 10$$

9. Deduce Lagrange's interpolation formula and also prove that Lagrangian functions are invariant under linear transformation. 8

- 10.(a) For any positive integer k , show that 4

$$\nabla^k y_n = \sum_{i=0}^k (-1)^i \binom{k}{i} y_{n-i}$$

∇ being the backward difference operator.

- (b) What do you mean by 'round off' errors in numerical data? Show how these errors are propagated in a difference table. 4

N.B. : Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.

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