



WEST BENGAL STATE UNIVERSITY
B.Sc. Honours/Programme 2nd Semester Examination, 2021

MTMHGEC02T/MTMGCOR02T-MATHEMATICS (GE2/DSC2)

Time Allotted: 2 Hours

Full Marks: 50

*The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.*

Answer Question No. 1 and any five from the rest

1. Answer any **five** questions from the following: 2×5 = 10

(a) Test whether the equation $x dx + y dy + \frac{x dy - y dx}{x^2 + y^2} = 0$ is exact or not.

(b) Find an integrating factor of the differential equation $(x \log x) \frac{dy}{dx} + y = 2 \log x$.

(c) Find particular integral of the differential equation $2x \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} = \frac{1}{x}$.

(d) Find the transformation of the differential equation $x^2 \frac{d^2 y}{dx^2} - 5y = \log x$, using the substitution $x = e^z$.

(e) Find complementary function of the differential equation $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} = 3x$.

(f) Find the Wronskian of $y_1(x) = e^{-2x}$, $y_2(x) = xe^{-2x}$.

(g) Construct a PDE by eliminating a and b from $z = ae^{-bt} \cos bx$.

(h) Determine the order, degree and linearity of the following PDE:

$$\frac{\partial z}{\partial x} = \left(\frac{\partial^2 z}{\partial x^2} \right)^{5/2} + \left(\frac{\partial^2 z}{\partial y^2} \right)^{5/2}$$

(i) Classify the following PDE

$$(1 + x^2) z_{xx} + (1 + y^2) z_{yy} + xz_x + yz_y = 0$$

into elliptic, parabolic and hyperbolic for different values of x and y .

2. (a) Find an integrating factor of the differential equation 4

$$(2xy^4 e^y + 2xy^3 + y) dx + (x^2 y^4 e^y - x^2 y^2 - 3x) dy = 0$$

and hence solve it.

- (b) Solve: $x \cos x \frac{dy}{dx} + y(x \sin x + \cos x) = 1$ 4
3. (a) Find the curve for which the area of the triangle formed by x -axis, a tangent and the radius vector of the point of tangency is constant and equal to a^2 . 4
- (b) Using the substitution $u = \frac{1}{x}$ and $v = \frac{1}{y}$, reduce the equation $y^2(y - px) = x^4 p^2$ to Clairaut's form and hence solve it. Here $p \equiv \frac{dy}{dx}$. 4
4. (a) Show that each of the functions e^x, e^{4x} and $2e^x - 3e^{4x}$ is solution of the differential equation $\frac{d^2 y}{dx^2} - 5\frac{dy}{dx} + 4y = 0, -\infty < x < \infty$. 2+1+1+1
Are the three independent? If not, find which two of these are independent. Write down a general solution of the equation.
- (b) Find the value of h so that the equation $(ax + hy + g) dx + (3x + by + f) dy = 0$ becomes an exact differential equation. 3
5. (a) Solve by the method of variation of parameters: 5
 $(D^2 - 3D + 2)y = e^x(1 + e^x)^{-1}$, where $D \equiv \frac{d}{dx}$
- (b) Find particular integral of the differential equation 3
 $(D^2 + 5D + 6)y = e^{-2x} \sin 2x$, where $D \equiv \frac{d}{dx}$
6. (a) Solve in the particular cases: 5
 $\frac{d^2 x}{dt^2} - 4\frac{dx}{dt} + 5x = 0$ giving that $x = 1$ and $\frac{dx}{dt} = 2$ when $x = 0$
- (b) Solve: $\frac{d^2 y}{dx^2} = x^2 \sin x$ 3
7. (a) Solve the following total differential equation: 4
 $yz dx + 2zx dy - 3xy dz = 0$
- (b) Solve: $x^2 \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} + y = x \log x$ 4
8. (a) Form a PDE by eliminating the arbitrary function ϕ from 4
 $lx + my + nz = \phi(x^2 + y^2 + z^2)$
- (b) Solve the partial differential equation by Lagrange's method $x^2 p + y^2 q = (x + y)z$. 4

9. (a) Find the partial differential equation of planes having equal intercepts along x axis and y axis. 4
- (b) Find $f(y)$ such that the total differential equation $\left(\frac{yz+z}{x}\right)dx - zdy + f(y)dz = 0$ is integrable. 4
- 10.(a) Formulate a PDE from the relation $f\left(\frac{x-a}{z-c}, \frac{y-b}{z-c}\right) = 0$. 3
- (b) Find the Wronskian of x and $|x|$ in $[-1, 1]$. 2
- (c) Solve $x^2 \frac{d^2y}{dx^2} - 6y = 0$. 3

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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