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Determination of Moment of Inertia of a Metallic Rectangular Bar about an Axis Passing Through its Centre of Gravity

- **Theory:**

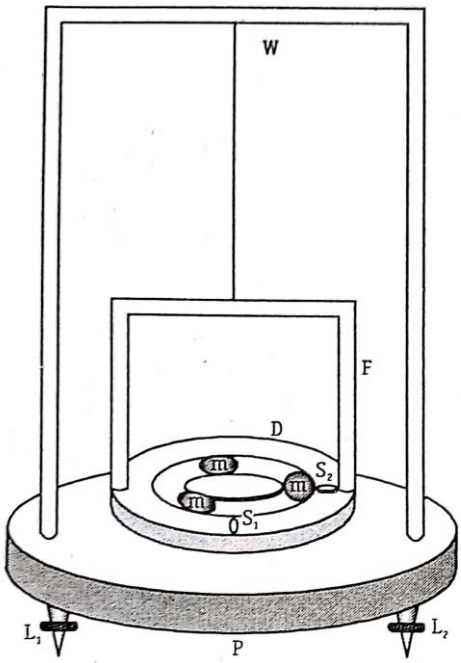

Figure: Instrument for determining moment of inertia

The time-period T of the torsional oscillation of a body is given by,

$$T = 2\pi\sqrt{\frac{I}{c}}$$

where, I is the moment of inertia and c represents the couple per unit twist in the suspension wire.

Now if the period of oscillation about the vertical axis,

i. of the cradle alone $= T_0$
ii. of the cradle and the body of known moment of inertia $= T_1$
iii. of the cradle and the body of unknown moment of inertia $= T_2$

Then,

$$T_0 = 2\pi\sqrt{\frac{I_0}{C}} \quad \ldots (1)$$

$$T_1 = 2\pi\sqrt{\frac{I_1+I_0}{C}} \quad \ldots (2)$$

$$T_2 = 2\pi\sqrt{\frac{I_2+I_0}{C}} \quad \ldots (3)$$

Where, I_0, I_1 & I_2 are respectively the moment of inertia of cradle, known body and unknown body.

Using equations (1), (2) and (3) we get

$$I_2 = I_1\frac{T_2^2-T_0^2}{T_1^2-T_0^2} \quad \ldots (4)$$

Now, the moment of inertia of a known rectangular bar about the vertical axis passing through its centre of gravity is given by

$$I_1 = \frac{M}{12}(l^2+b^2) \quad \ldots (5)$$

where, *M*, *l* and *b* represent the mass, length and breadth of the known body respectively.

Similarly, the moment of inertia of an unknown rectangular bar about the vertical axis passing through its centre of gravity is given by,

$$I'_2 = \frac{M'}{12}(l'^2+b'^2) \quad \ldots (6)$$

where, *M'*, *l'* and *b'* represent the mass, length and breadth of the unknown body respectively.

Moment of inertia of an unknown rectangular bar is to be determined using equation (4); also, using equation (6), moment of inertia of an unknown rectangular bar is to be determined and these two values are to be compared. If the value of $\frac{I_2}{I'_2}$ approaches unity (1), experimental results is said to be correct.

- **Experimental Results:**

(A) Determination of the Moment of Inertia (I_1) of a Known Rectangular Bar:

(i) Mass of the known body = M = ... *Kg*

(ii) Determination of Vernier constant:

1 small division of main scale = *0.1 cm*
Now, 10 vernier divisions = 9 small main scale divisions
or, 1 v.d. = $\frac{9}{10}$ s.m.d.
So, Vernier constant (*v*) = 1 small main scale division – 1 vernier division
= $(1-\frac{9}{10})$ x 0.1 *cm* = *0.01 cm*

(iii) Determination of mechanical error (e) of Slide Callipers:

$e = \pm y \times v$ *cm* [where y is the initial reading of erroneous slide calipers]

Table – I

Determination of Length (*l*) of the Rectangular Bar using Slide Callipers

No. of Observation	Main Scale Reading (S) (in *cm*)	Vernier Scale Reading (N)	Total Reading $Total = S + N \times v$	Mean l_1 (in *cm*)	Corrected length $l = l_1 \pm e$ (in *cm*)
1					
2					
3					

Table – II

Determination of Breadth (*b*) of the Rectangular Bar using Slide Callipers

No. of Observation	Main Scale Reading (S) (in *cm*)	Vernier Scale Reading (N)	Total Reading $Total = S + N \times v$	Mean b_1 (in *cm*)	Corrected length $b = b_1 \pm e$ (in *cm*)
1					
2					
3					
4					
5					

From equation (5), $I_1 = \frac{M}{12}(l^2 + b^2)$ = ... *kg-m*^{2}

(B) Determination of the Time-period:

Table – III

Oscillating Body	Time for 30 oscillations (*t*) (in *s*)	Mean time (*t*) (in *s*)	Time-period $\frac{t}{30}$ (in *s*)
Cradle only	1. 2. 3.		… (T_0)
Cradle and known body	1. 2. 3.		… (T_1)
Cradle and unknown body	1. 2. 3.		… (T_2)

From equation (4), $I_2 = I_1 \frac{T_2^2 - T_0^2}{T_1^2 - T_0^2}$ = ... *Kg-m*^{2}

(C) Determination of the moment of inertia (I'_2) of unknown rectangular bar:

Mass of the unknown body = M´ = ... *K*g

Table – IV

Determination of length (l') of the rectangular bar using Slide Callipers

No. of Observation	Main Scale Reading (S) (in *cm*)	Vernier Scale Reading (N)	Total Reading $Total = S + N \times v$	Mean l'' (in *cm*)	Corrected length $l' = l'' \pm e$ (in *cm*)
1					
2					
3					

Table – V

Determination of length (b') of the rectangular bar using Slide Callipers

No. of Observation	Main Scale Reading (S) (in *cm*)	Vernier Scale Reading (N)	Total Reading $Total = S + N \times v$	Mean b'' (in *cm*)	Corrected length $b' = b'' \pm e$ (in cm)
1					
2					
3					
4					
5					

From equation (6), $I'_2 = \frac{M'}{12}\left(l'^2 + b'^2\right)$= ..... kg-m^2

(D) Determination of the moment of inertia (I'_2) of unknown rectangular bar theoretically and comparison with the experimental value (I'_2)

Table – VI

Value (I_2) obtained from Table-III (in Kg-m^2)	**Value** (I'_2) obtained from Table-V (in Kg-m2)	$\frac{I_2}{I'_2}$