The Special Theory of Relativity

(NEP Semester V - Chapter 1)

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Newtonian (Classical) Relativity

ASSUMPTION

- IT IS ASSUMED THAT NEWTON'S LAWS OF MOTION MUST BE MEASURED WITH RESPECT TO (RELATIVE TO) SOME REFERENCE FRAME.

Newtonian Principle of Relativity

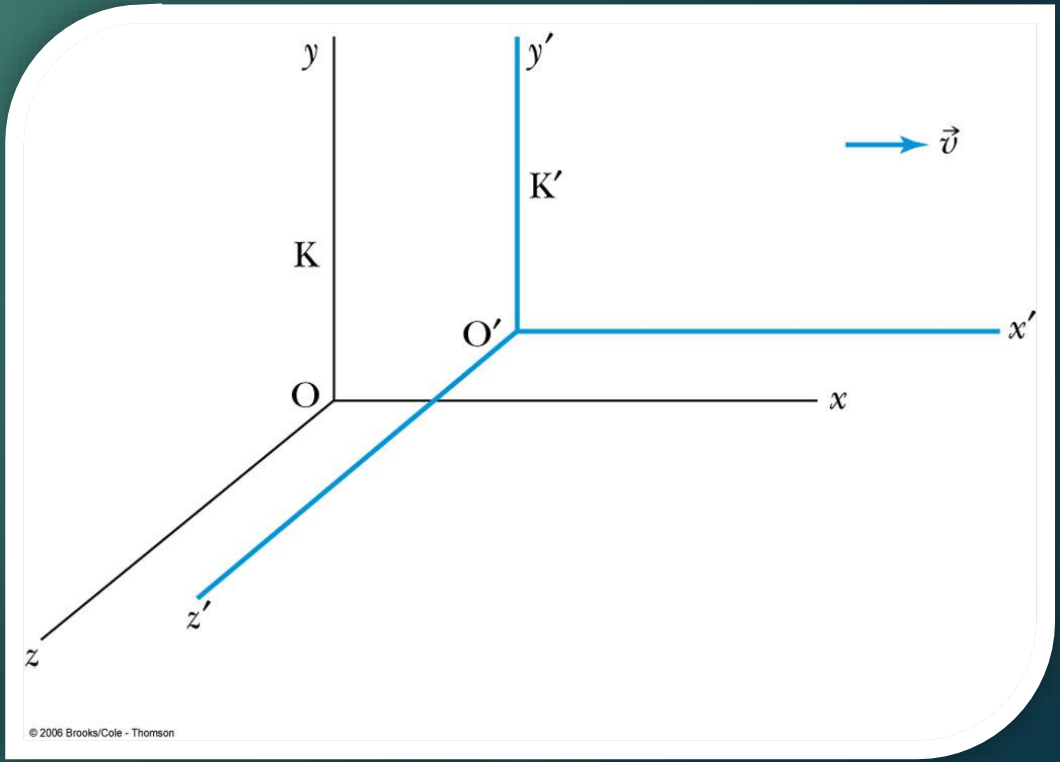
- If Newton's laws are valid in one reference frame, then they are also valid in another reference frame moving at a uniform velocity relative to the first system.

- This is referred to as the **Newtonian principle of relativity** or **Galilean invariance.**

Inertial Reference Frame

- A reference frame is called an **inertial frame** if Newton laws are valid in that frame.
- Such a frame is established when a body, not subjected to net external forces, is observed to move in rectilinear motion at constant velocity.

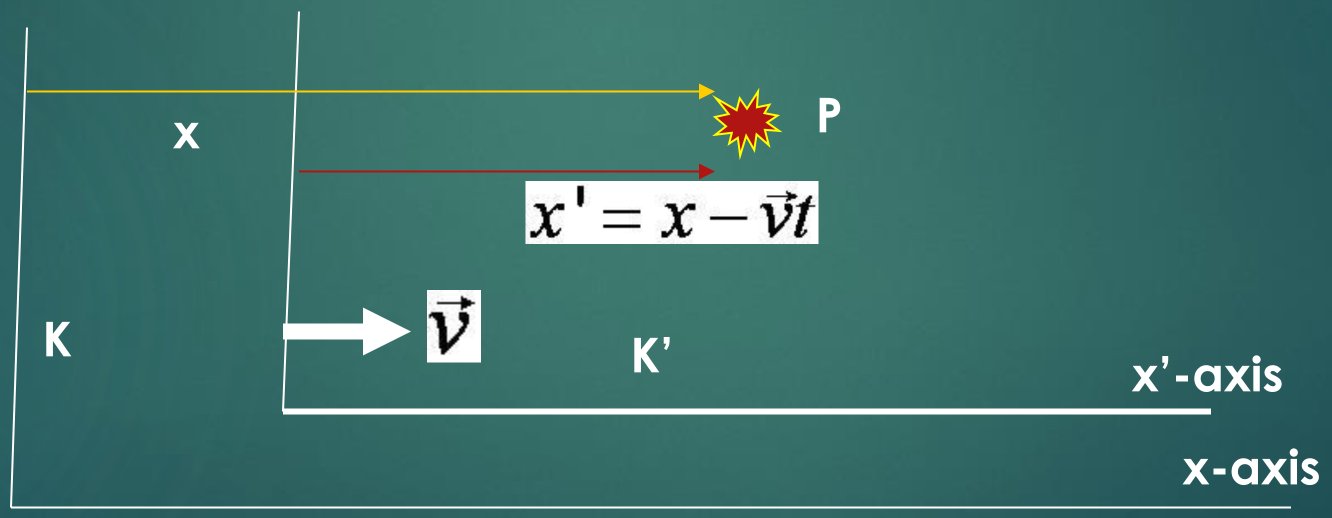
K is at rest and K' is moving with velocity $\vec{v}$
All Axes are parallel
K and K' frames are said to be
INERTIAL COORDINATE SYSTEMS



The Galilean Transformation

For a point P

- In system K: P = (x, y, z, *t*)
- In system K': P = (x', y', z', *t*')



Conditions of the Galilean Transformation

- Parallel axes
- K' frame has a constant relative velocity along the x-direction with respect to K frame

$$x' = x - \vec{v}t$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

- ***Time*** (*t*) for all observers is a *Fundamental invariant*, i.e., the same for all inertial observers

The Inverse Relations

Step 1. Replace $\vec{v}$ with $-\vec{v}$

Step 2. Replace "primed" quantities with "unprimed" and "unprimed" with "primed."

$$x = x' + \vec{v}t$$

$$y = y'$$

$$z = z'$$

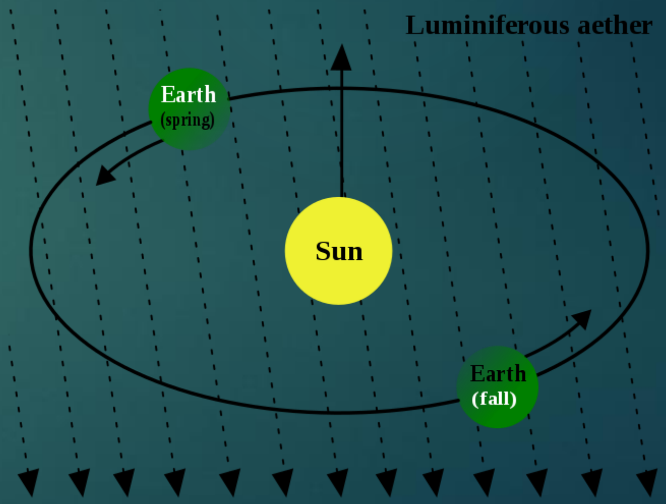
$$t = t'$$

The Transition to Modern Relativity

- Although Newton's laws of motion had the same form under the Galilean transformation, Maxwell's equations did not.
- In 1905, Albert Einstein proposed a fundamental connection between space and time and that Newton's laws are only an approximation.

The Need for Ether

- The wave nature of light suggested that there existed a propagation medium called the ***luminiferous ether*** or just ***Ether***.
- Ether had to have such a low density that the planets could move through it without loss of energy.
- It also had to have an elasticity to support the high velocity of light waves



A depiction of the concept of the "aether wind"

Maxwell's Equations

- In Maxwell's theory the speed of light, in terms of the permeability and permittivity of free space, was given by

$$v = c = 1/\sqrt{\mu_0 \varepsilon_0}$$

- Thus the velocity of light between moving systems must be a constant.

An Absolute Reference System

- Ether was proposed as an absolute reference system in which the speed of light was this constant and from which other measurements could be made.
- The Michelson-Morley experiment was an attempt to show the existence of ether.

The Michelson-Morley Experiment

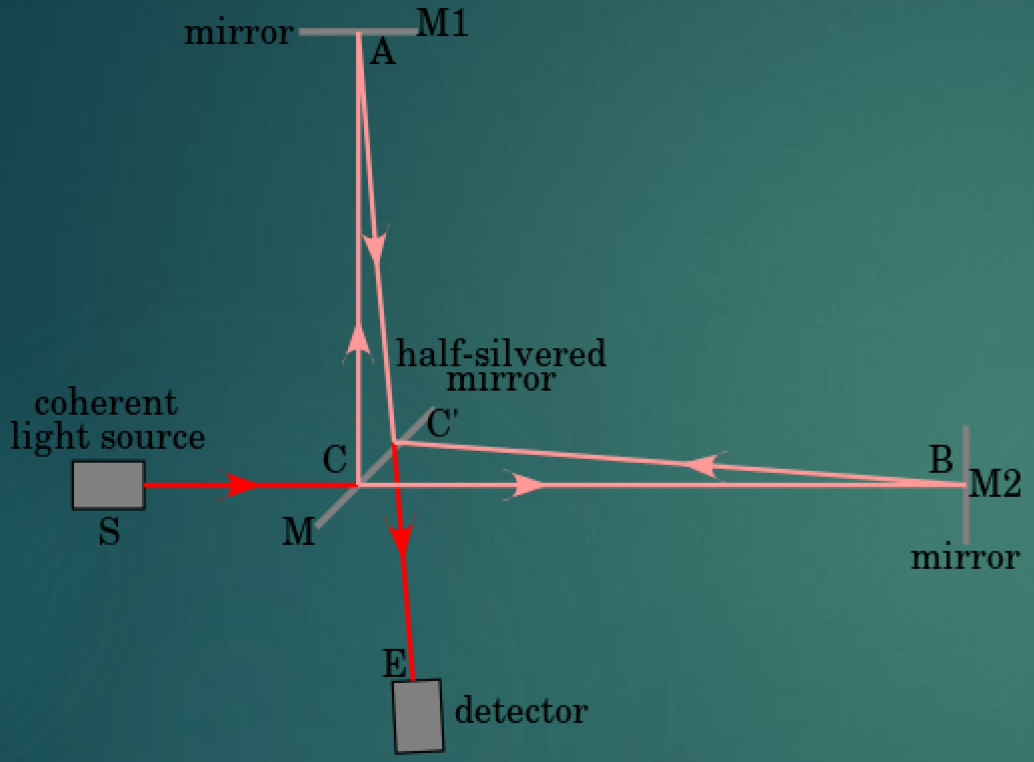
Albert Michelson
(1852–1931)

Edward Williams Morley
(1838 –1923)

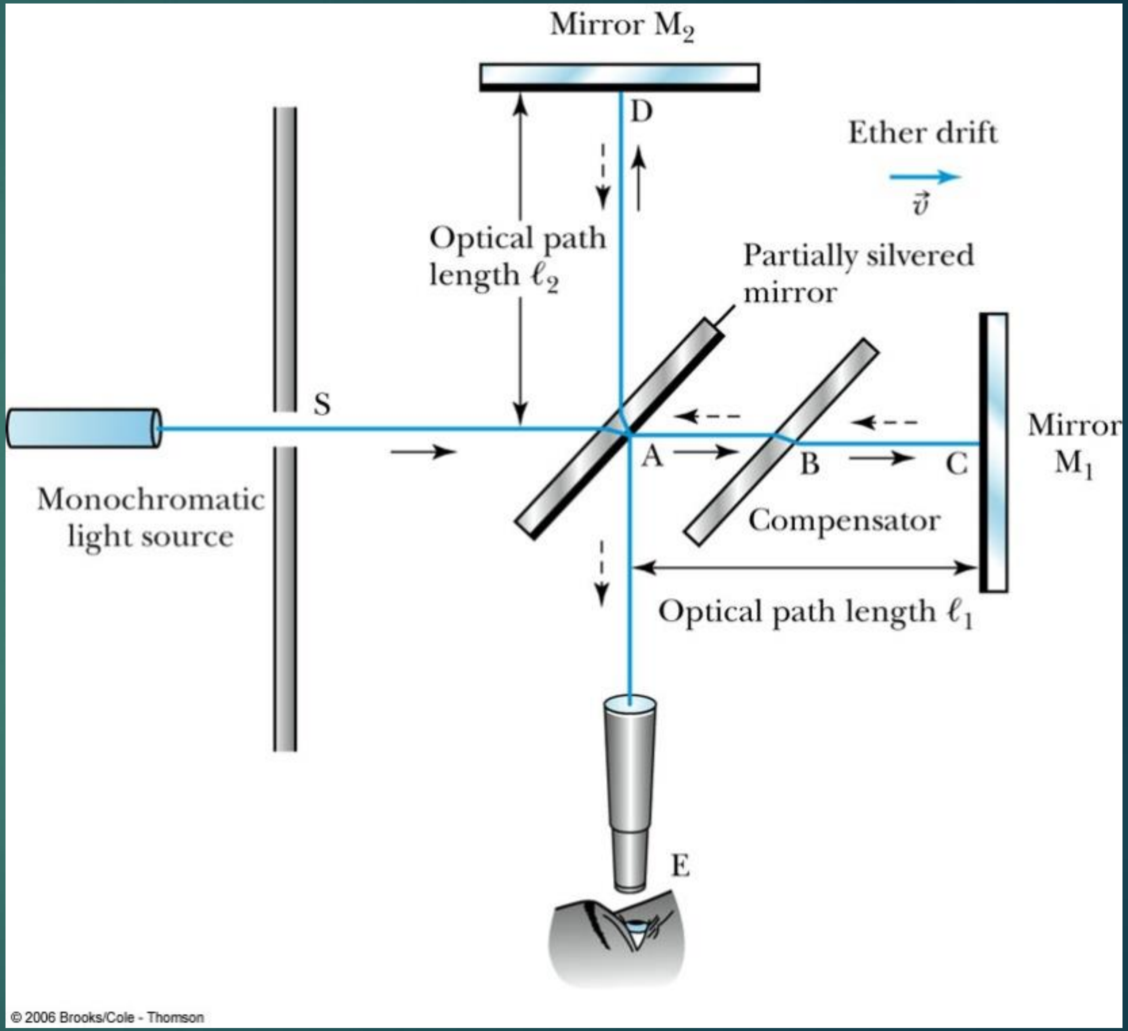
- The Michelson–Morley experiment was an attempt to detect the existence of the luminiferous aether.
- The experiment was performed between April and July 1887 by Albert A. Michelson and Edward W. Morley.
- The experiment compared the speed of light in perpendicular directions in an attempt to detect the relative motion of matter through the stationary luminiferous aether ("aether wind").

Most famous "failed" experiment

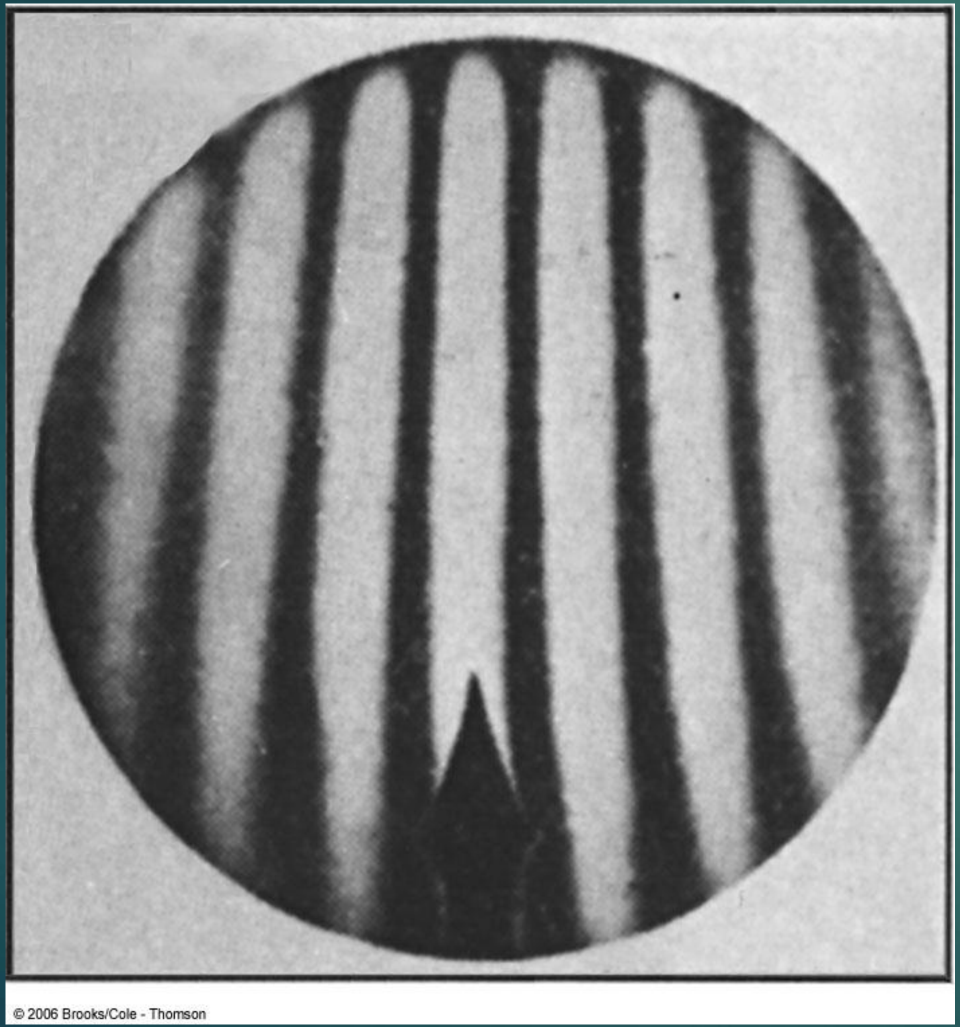
The Michelson Interferometer

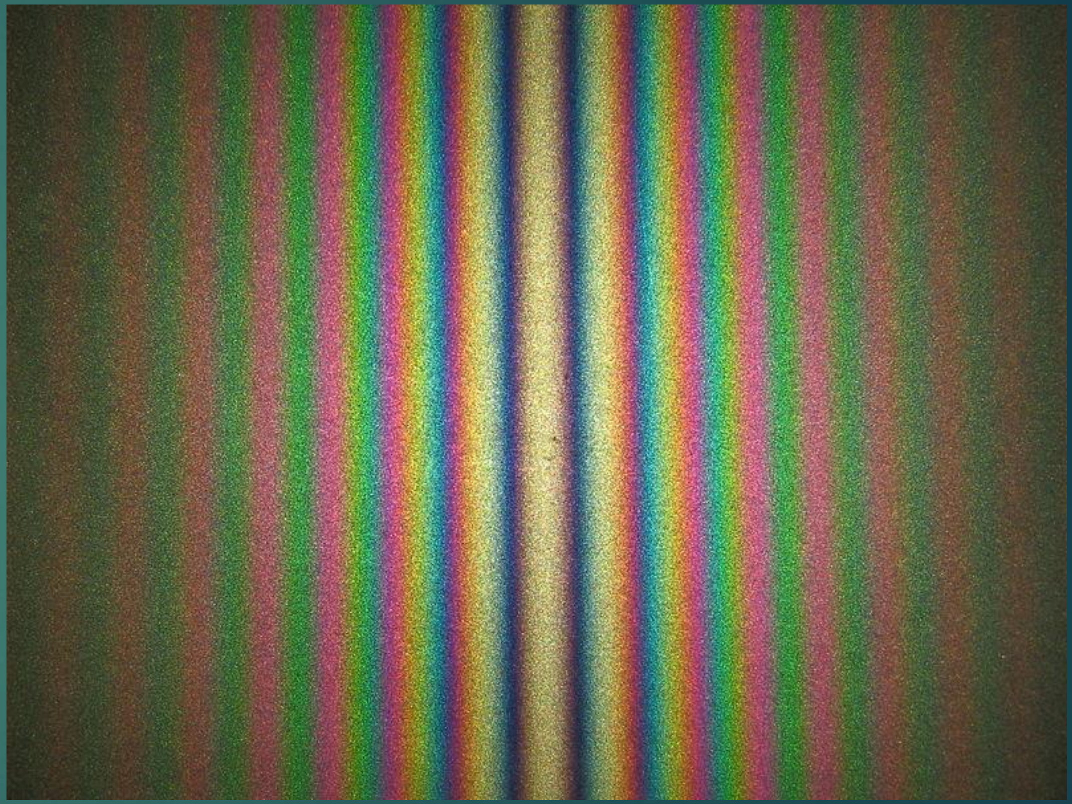


Path of light in Michelson interferometer



Typical interferometer fringe pattern expected when the system is rotated by 90°





Michelson's Conclusion

- Michelson noted that he should be able to detect a phase shift of light due to the time difference between path lengths but found none.
- He thus concluded that the hypothesis of the stationary ether must be incorrect.
- After several repeats and refinements with assistance from Edward Morley (1893-1923), again *a null result*.
- ***Thus, ether does not seem to exist!***

The Lorentz-FitzGerald Contraction

- Another hypothesis proposed independently by both H. A. Lorentz and G. F. FitzGerald suggested that the length ℓ_1, in the direction of the motion was *contracted* by a factor of

$$\sqrt{1 - v^2 / c^2}$$

…thus making the path lengths equal to account for the zero phase shift.

- This, however, was an ad hoc assumption that could not be experimentally tested.

Einstein's Postulates

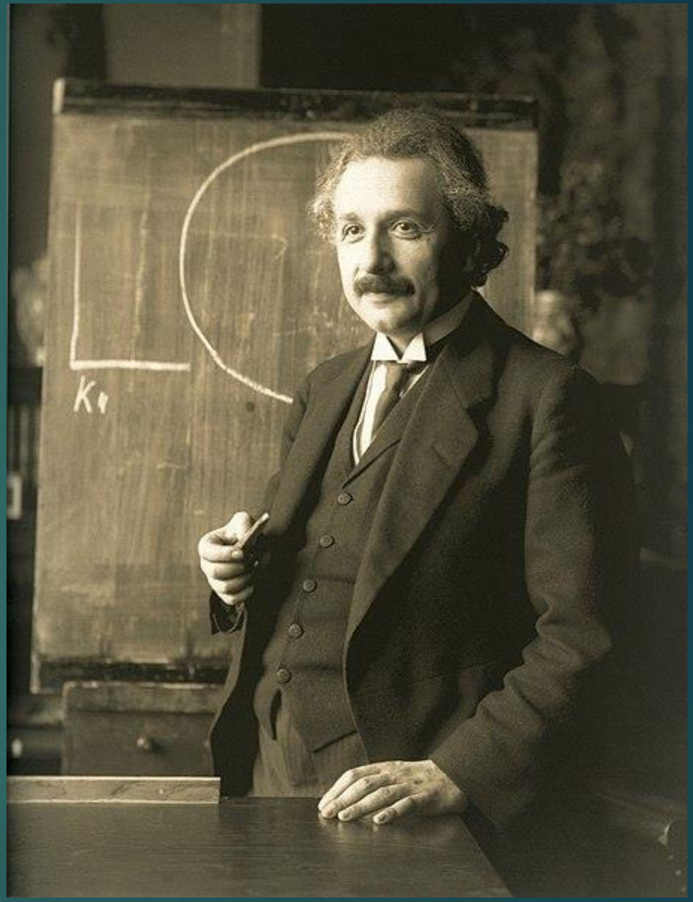
With the belief that Maxwell's equations must be valid in all inertial frames, Einstein proposes the following postulates:

1) **The principle of relativity**

The laws of physics are the same in all inertial systems. There is no way to detect absolute motion, and no preferred inertial system exists.

1) **The constancy of the speed of light**

Observers in all inertial systems measure the same value for the speed of light as in a vacuum.



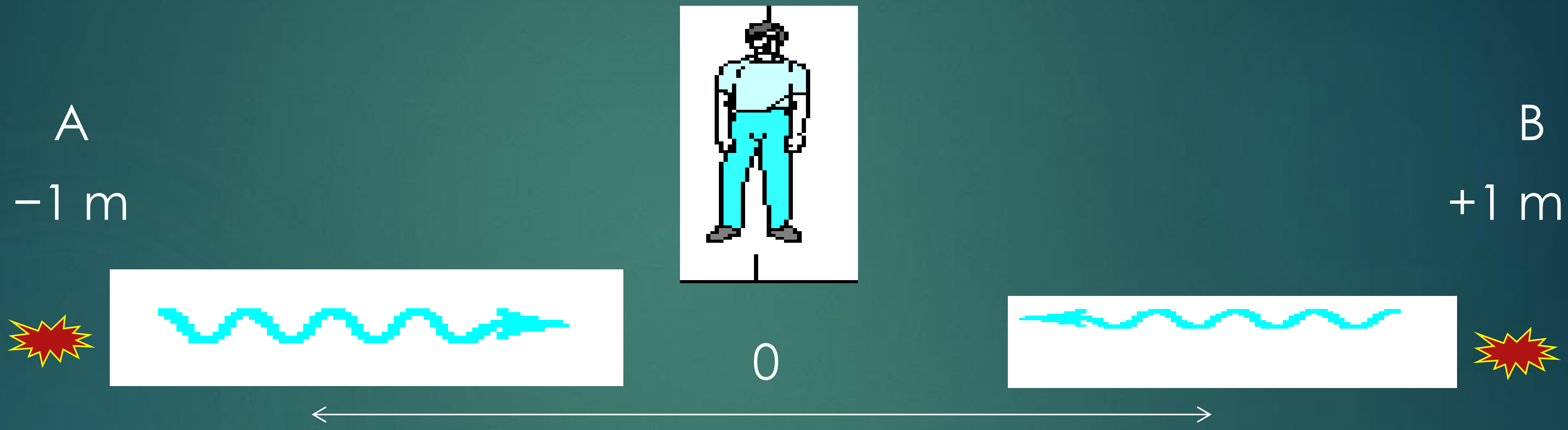
Albert Einstein (1879–1955)

Re-evaluation of Time

- In Newtonian physics we previously assumed that $t = t'$
  - *Thus with "synchronized" clocks, events in K and K' can be considered simultaneous*
- Einstein realized that each system must have its own observers with their own clocks and meter sticks
  - Thus, events considered simultaneous in K may not be in K'

The Problem of Simultaneity

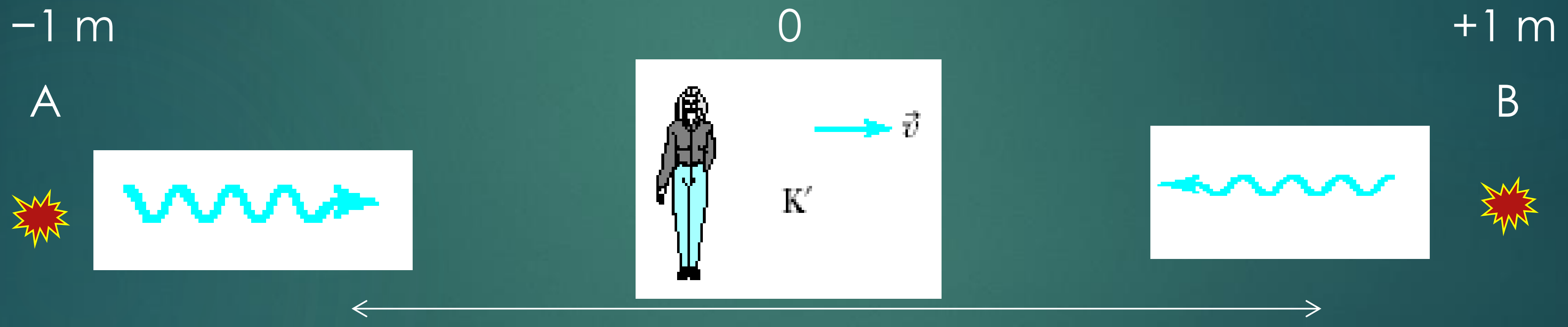
Frank at rest is equidistant from events A and B:



Frank “sees” both flashbulbs go off simultaneously.

The Problem of Simultaneity

Mary, moving to the right with speed v, observes events A and B in different order:



Mary "sees" event B, then A.

We thus observe…

- *Two events that are simultaneous in one reference frame (K) are not necessarily simultaneous in another reference frame (K') moving with respect to the first frame.*

- This suggests that each coordinate system has its own observers with "clocks" that are synchronized…

The Lorentz Transformations

The special set of linear transformations that:

1) preserve the constancy of the speed of light (c) between inertial observers;

and,

2) account for the problem of simultaneity between these observers

known as the **Lorentz transformation equations**

Lorentz Transformation Equations

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - (vx/c^2)}{\sqrt{1 - v^2/c^2}}$$

Suppose,

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Then,

$$x' = \gamma(x - \beta ct)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma(t - \beta x/c)$$

The complete Lorentz Transformation Equations

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - (vx/c^2)}{\sqrt{1-\beta^2}}$$

$$x = \frac{x' + vt'}{\sqrt{1-\beta^2}}$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1-\beta^2}}$$

Time Dilation and Length Contraction

Consequences of the Lorentz Transformation:

- **Time Dilation**:

  Clocks in K' run slow with respect to stationary clocks in K.

- **Length Contraction**:

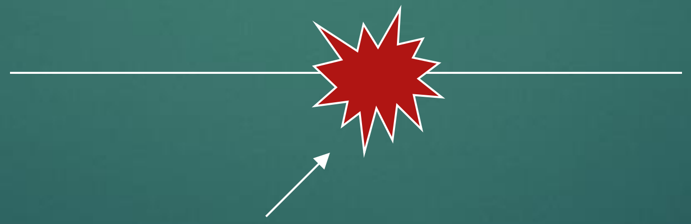
  Lengths in K' are contracted with respect to the same lengths stationary in K.

Proper Time

To understand *time dilation* the idea of **proper time** must be understood:

- The term **proper time**,T_0, is the time difference between two events occurring at the same position in a system as measured by a clock at that position.

Same location

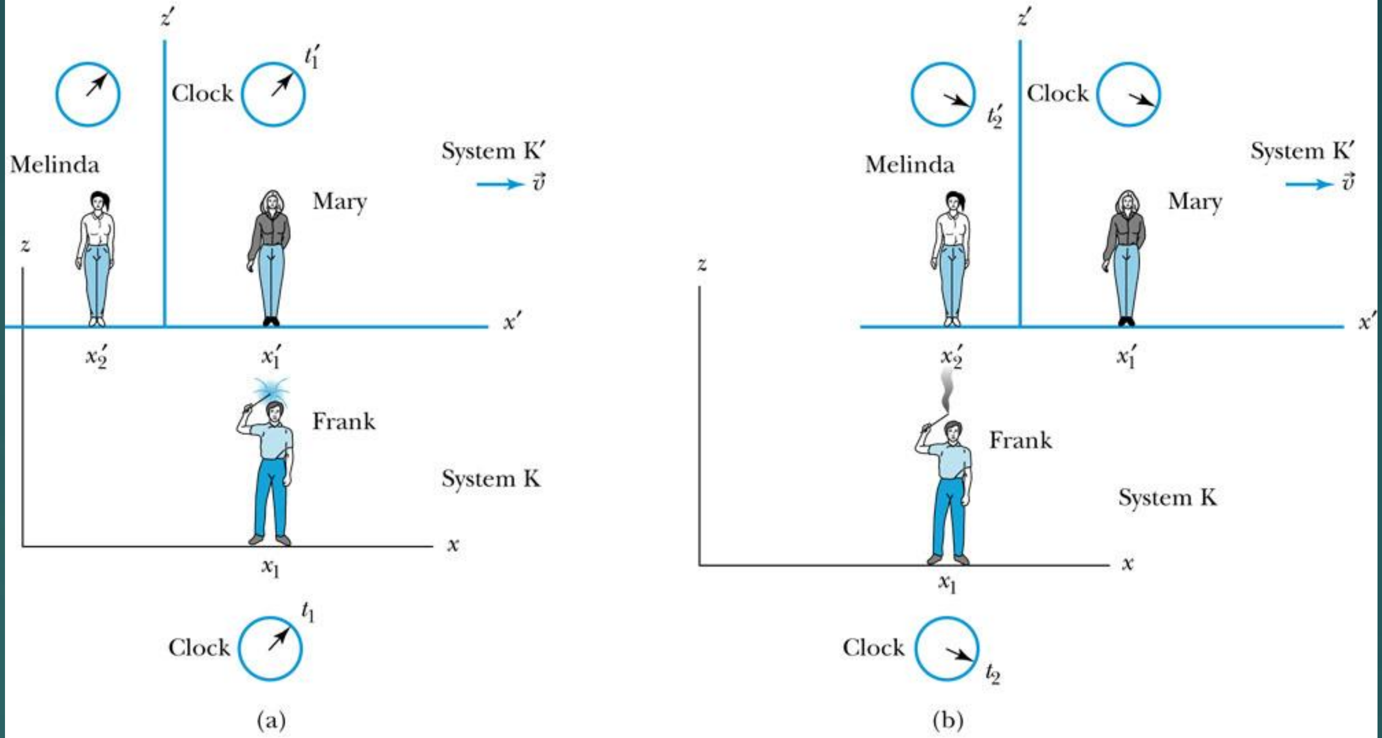


Proper Time

Not Proper Time

Beginning and ending of the event occur at different positions

Time Dilation



Frank's clock is at the same position in system K when the sparkler is lit in (a) and when it goes out in (b). Mary, in the moving system K', is beside the sparkler at (a). Melinda then moves into the position where and when the sparkler extinguishes at (b). Thus, Melinda, at the new position, measures the time in system K' when the sparkler goes out in (b).

According to Mary and Melinda…

- Mary and Melinda measure the two times for the sparkler to be lit and to go out in system K' as times t'_1 and t'_2 so that by the Lorentz transformation:

$$t'_2 - t'_1 = \frac{(t_2 - t_1) - (v/c^2)(x_2 - x_1)}{\sqrt{1 - v^2/c^2}}$$

  - Note here that Frank records $x - x_1 = 0$ in K with a proper time: $T_0 = t_2 - t_1$ or

$$T' = \frac{T_0}{\sqrt{1 - v^2/c^2}} = \gamma T_0$$

with $T' = t'_2 - t'_1$

Time Dilation

1) $T' > T_0$ or the time measured between two events at *different positions* is greater than the time between the same events at *one position*: ***time dilation.***
2) The events do not occur at the same space and time coordinates in the two system
3) System K requires 1 clock and K' requires 2 clocks.

Length Contraction: Proper Length

To understand *length contraction* the idea of **proper length** must be understood:

- Let an observer in each system K and K’ have a meter stick at rest in their own system such that each measure the same length at rest.
- The length as measured at rest is called the **proper length**.

What Frank and Mary see…

Each observer lays the stick down along his or her respective x axis, putting the left end at x_ℓ (or x'_ℓ) and the right end at x_r (or x'_r).

- Thus, in system K, Frank measures his stick to be:

$$L_0 = x_r - x_\ell$$

- Similarly, in system K', Mary measures her stick at rest to be:

$$L'_0 = x'_r - x'_\ell$$

What Frank and Mary measure

- Frank in his rest frame measures the moving length in Mary's frame moving with velocity.

- Thus using the Lorentz transformations Frank measures the length of the stick in K' as:

$$x'_r - x'_\ell = \frac{(x_r - x_\ell) - v(t_r - t_\ell)}{\sqrt{1 - v^2/c^2}}$$

Where both ends of the stick must be measured simultaneously, i.e, $t_r = t_\ell$

Here Mary's proper length is $L'_0 = x'_r - x'_\ell$

and Frank's measured length is $L = x_r - x_\ell$

Frank's measurement

So Frank measures the moving length as *L* given by

$$L'_0 = \frac{L}{\sqrt{1-v^2/c^2}} = \gamma L$$

but since both Mary and Frank in their respective frames measure $L'_0 = L_0$

$$L = L_0\sqrt{1-v^2/c^2} = \frac{L_0}{\gamma}$$

and $L_0 > L$, i.e. the moving stick shrinks.