

Diffraction of Light

(NEP Semester III - Chapter 6)

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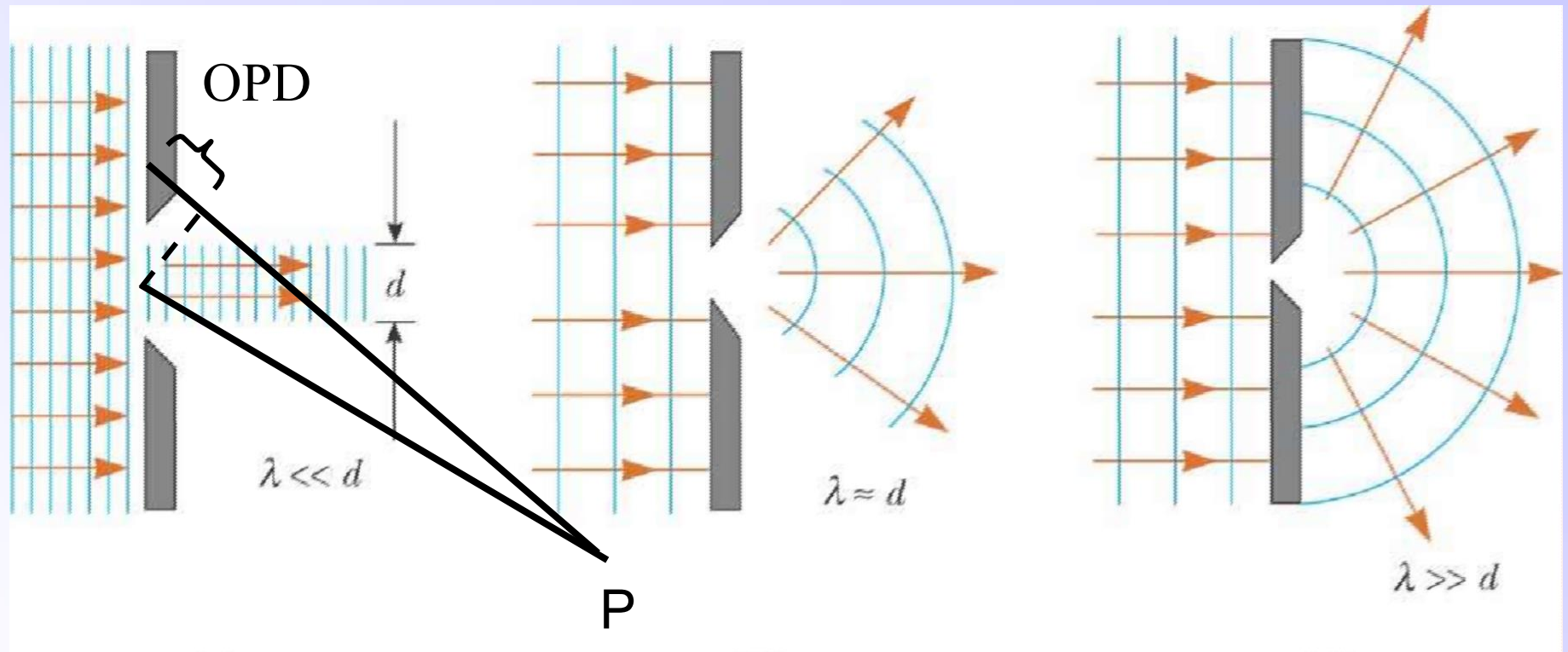
- Diffraction
 - It refers to deviation of light from rectilinear propagation.
 - It is a general characteristics of a wave phenomena occurring whenever a portion of a wavefront is *obstructed* in some way.
- There is no significant distinction between interference and diffraction.
 - It is customary to speak of interference when considering the superposition of only a few waves; and diffraction when treating a large number of waves.
- The classical wave theory with its simple and effective formalism will be more than sufficient to describe diffraction.

- *Huygen-Fresnel principle:*

Every unobstructed point of a wavefront, at a given instant, serves as a source of spherical secondary wavelets (with the same frequency as that of the primary wave).

The amplitude of the optical field at any point beyond is the superposition of all these wavelets (considering their amplitudes and relative phases).

- Fresnel resolves the inadequacy of Huygen's principle with the addition of the concept of interference.



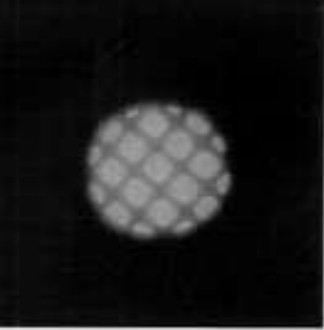
- Each unobstructed point act as a secondary source.
- Consider an arbitrary point, P , far away from the obstruction. The maximum path difference among all those sources are due to the two by the sides, OPD .

Fraunhofer and Fresnel Diffraction

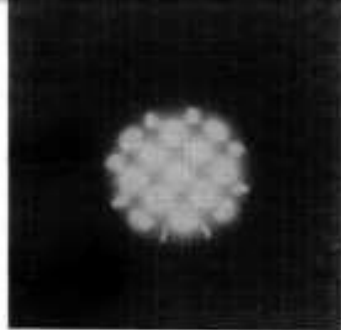
- Diffraction phenomena can be classified either as Fraunhofer diffraction or Fresnel diffraction.
- The observable difference :
Fresnel diffraction
 - The viewing screen and the aperture are located close together, the image of the aperture is clearly recognisable despite slight fringing around its periphery.
 - As the separation between the screen and the aperture increases, the image of the aperture becomes increasingly more structured; fringes become more prominent.

Fraunhofer diffraction

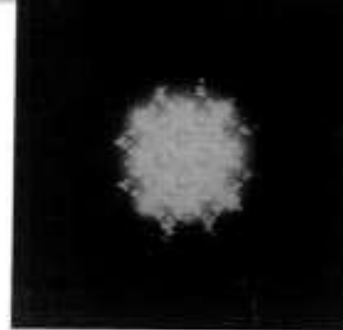
- The viewing screen and the aperture separated by a large distance, the projected pattern bears little or no resemblance to the aperture.
- As the separation increases, the size of the pattern changes but not its shape.



(a)



(b)



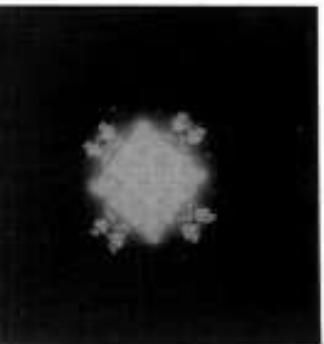
(c)

(a) : Aperture –
array type

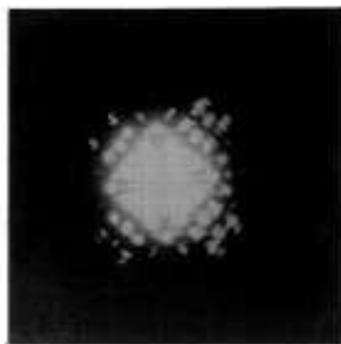
(b)-(h):

Fresnel diffraction
pattern

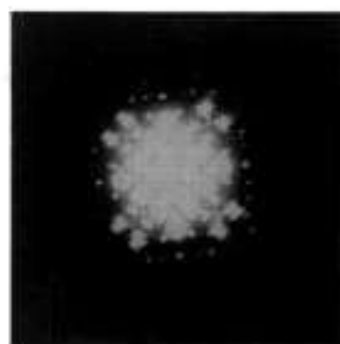
(note the changes
in the pattern as
the screen moves
further away)



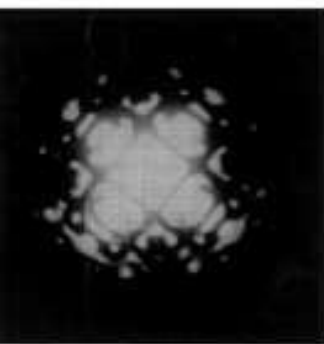
(d)



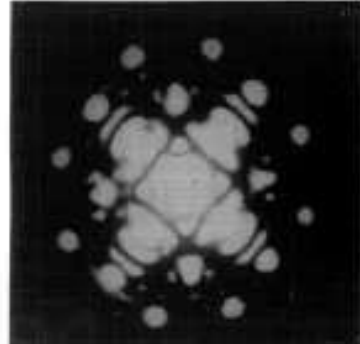
(e)



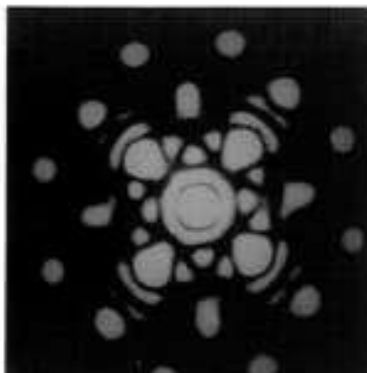
(f)



(g)



(h)



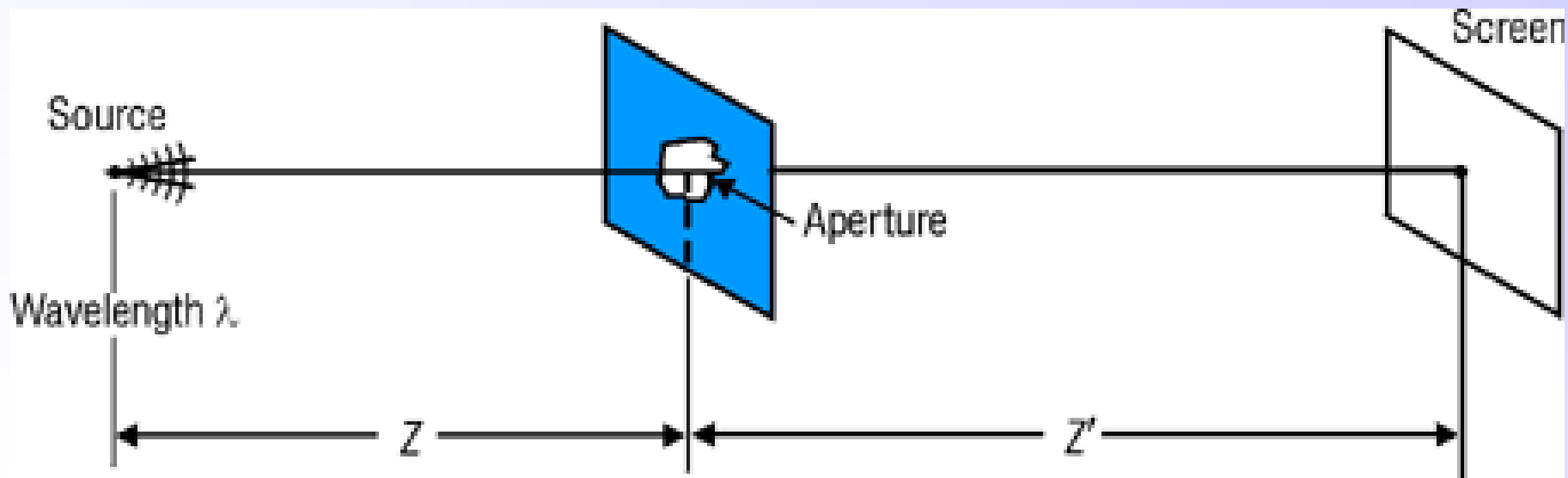
(i)

(i) : Fraunhofer
diffraction

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Fraunhofer and Fresnel Diffraction

- Consider the following setup:
 - An opaque screen, Σ , contains a single small aperture.
 - The aperture is illuminated by plane waves from a very distant source, S .
 - The plane of observation, σ , is a screen parallel to and located very close to Σ .

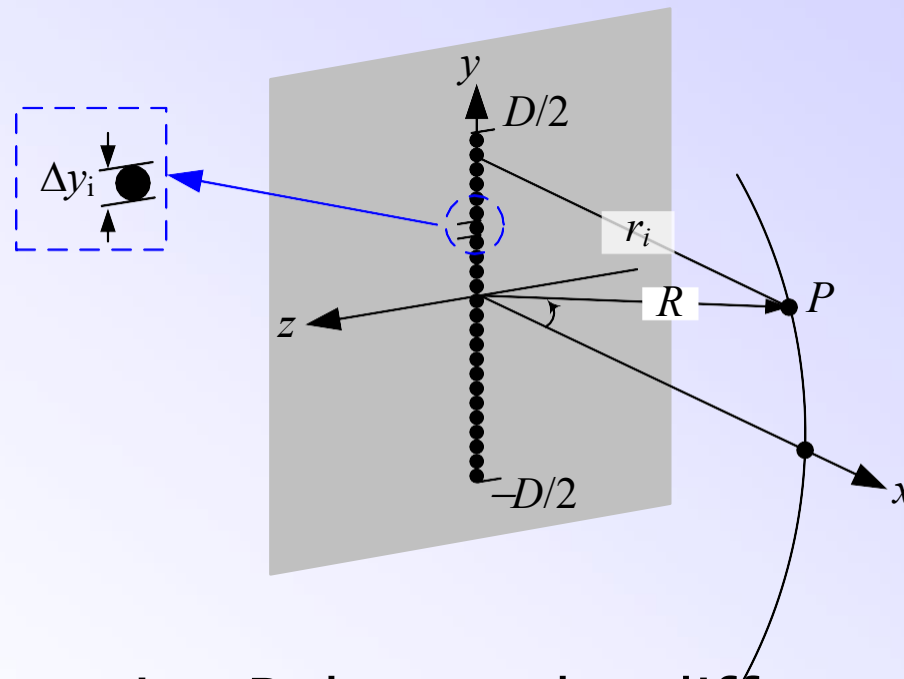


- The wavefronts impinging and emerging from the aperture also play a role in determining the type of diffraction which occurs:
 - If both waves are planar over the extent of the aperture → Fraunhofer diffraction
 - Phase difference from each source at point P on the screen is due to differences in the path traversed.
 - Path differences can then be described by a linear function of the two aperture variables.
 - The curvature of the waves are not negligible → Fresnel diffraction.
 - When S is nearby the aperture, the strength of the electric field to each point on the aperture will be different (it drops off inversely with distance).
 - The same consideration (differences in field strength), applies to the emerging rays directed at the observation screen.

Fraunhofer Diffraction

Single Slit

- Consider the figure below, the observation point is very far from the coherent line source and $R \gg D$.



- The field at point P due to the differential segment of the source dy is

$$dE = \frac{E_L}{R} \sin(\omega t - kr) dy$$

- At this point, the expression above is very sensitive to r where it can be expanded to

$$r = R - y \sin \theta + (y^2 / 2R) \cos^2 \theta + \dots$$

- Dropping the 3rd term leaves us with the **Fraunhofer condition** where the distance r is linear in y .
 - The distance to the point of observation and the phase can be written as a linear function of the aperture variables.
- Integrating for all differential segments of the coherent line source,

$$E = \frac{E_L D}{R} \frac{\sin \left[\left(\frac{kD}{2} \right) \sin \theta \right]}{\left(\frac{kD}{2} \right) \sin \theta} \sin(\omega t - kR)$$

$$E = \frac{E_L D}{R} \frac{\sin \beta}{\beta} \sin(\omega t - kR)$$

where $\beta = \left(\frac{kD}{2} \right) \sin \theta$

- The irradiance at point P ,

$$I(\theta) = \langle E^2 \rangle_T$$

$$I(\theta) = \frac{1}{2} \left(\frac{E_L D}{R} \right)^2 \left(\frac{\sin \beta}{\beta} \right)^2$$

- At $y = 0$ at the observation screen, $\theta = 0$; $\frac{\sin \beta}{\beta} = 1$ and $I(\theta) = I(0) = \frac{1}{2} \left(\frac{E_L D}{R} \right)^2$.

- Taking into consideration this condition, the irradiance can be written as

$$I(\theta) = I(0) \left(\frac{\sin \beta}{\beta} \right)^2$$

- There is symmetry about the y -axis, and the expression is correct for θ measured in any plane containing that axis.

- Since $\beta = \left(kD/2\right)\sin\theta = \left(\pi D/\lambda\right)\sin\theta$
when $D \gg \lambda$, the irradiance drops rapidly as θ deviates from zero.
- Now, a physical slit is *different* from that of a coherent line source.
 - A slit can have a width of several hundred λ and a length of a few centimeters.
- In order to generalize the coherent line source to that of a slit, divide the slit into a series of long differential strips of dimension dz by l parallel to the y -axis.
 - Each strip represents a long coherent line source and can be replaced by a point source along z -axis.

- It can be shown that the complete solution for the slit is equivalent to that of a line source, which leaves us with the irradiance of

$$I(\theta) = I(0) \left(\frac{\sin \beta}{\beta} \right)^2$$

where $\beta = \left(\frac{kb}{2} \right) \sin \theta = \left(\frac{\pi b}{\lambda} \right) \sin \theta$ and b is the width of the slit.

- Let $dI/d\beta = 0$; then $I(0) \frac{2 \sin \beta (\beta \cos \beta - \sin \beta)}{\beta^3} = 0$
- The irradiance has **minima** equal to zero when $\sin \beta = 0$ i.e.

$$\beta = \pm \pi; \pm 2\pi; \pm 3\pi; \dots$$

$$\left(\frac{\pi b}{\lambda} \right) \sin \theta_m = m\pi; \quad m = 1, 2, 3, \dots$$

$$\underline{b \sin \theta_m = m\lambda}$$

$$I(\theta) = I(0) \left(\frac{\sin \beta}{\beta} \right)^2$$

- Consider the following condition:

- $\beta = 0, \pm m\pi; \quad m = 1, 2, 3, \dots$

- $\beta = 0$: $\left(\frac{\sin \beta}{\beta} \right)^2 \rightarrow 1$
 $I(\theta) = I(0)$

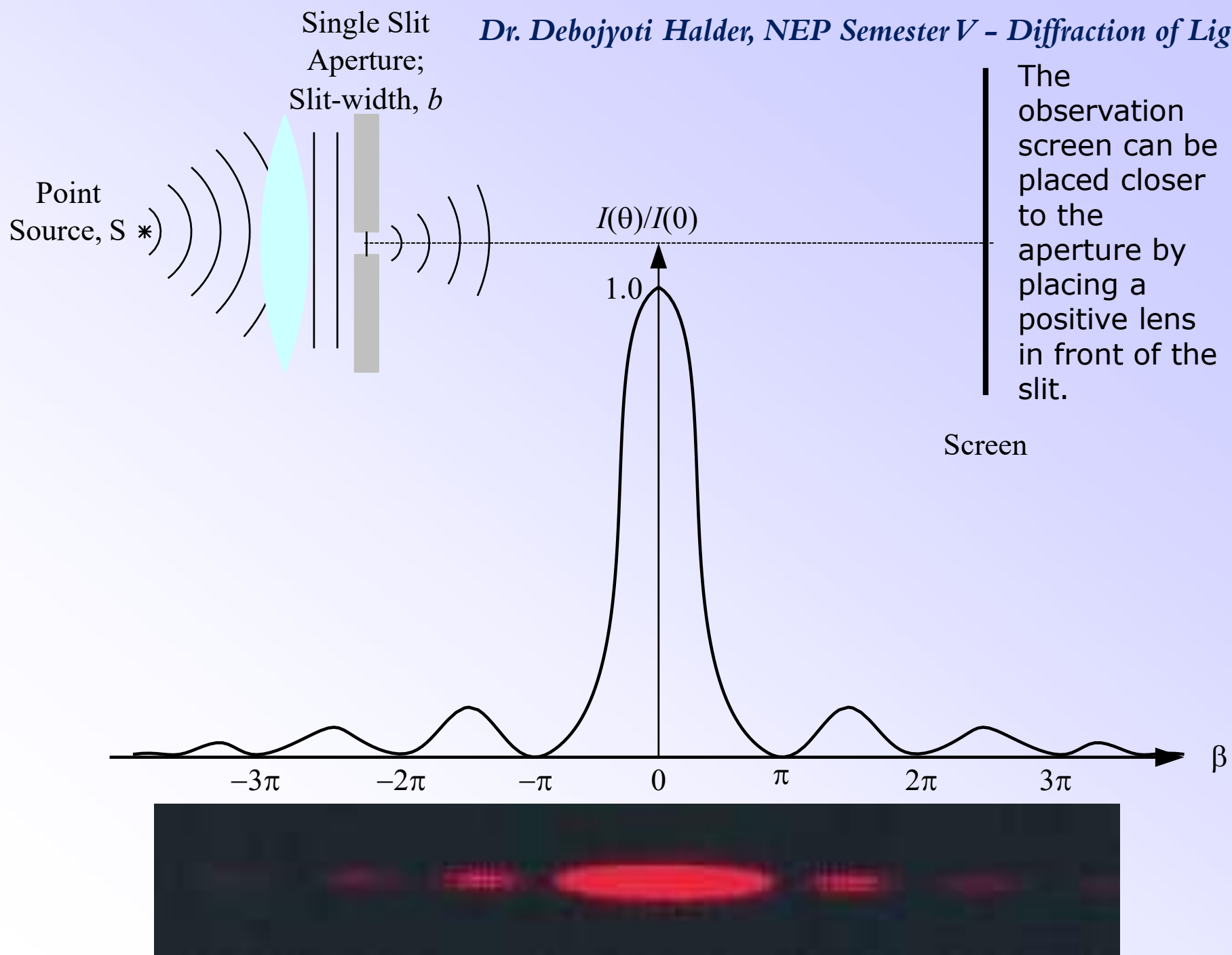
- $\beta = \pm m\pi$; $m = 1, 2, 3, \dots$
 $\sin \beta = 0 \quad \rightarrow \quad I(\theta) = 0$

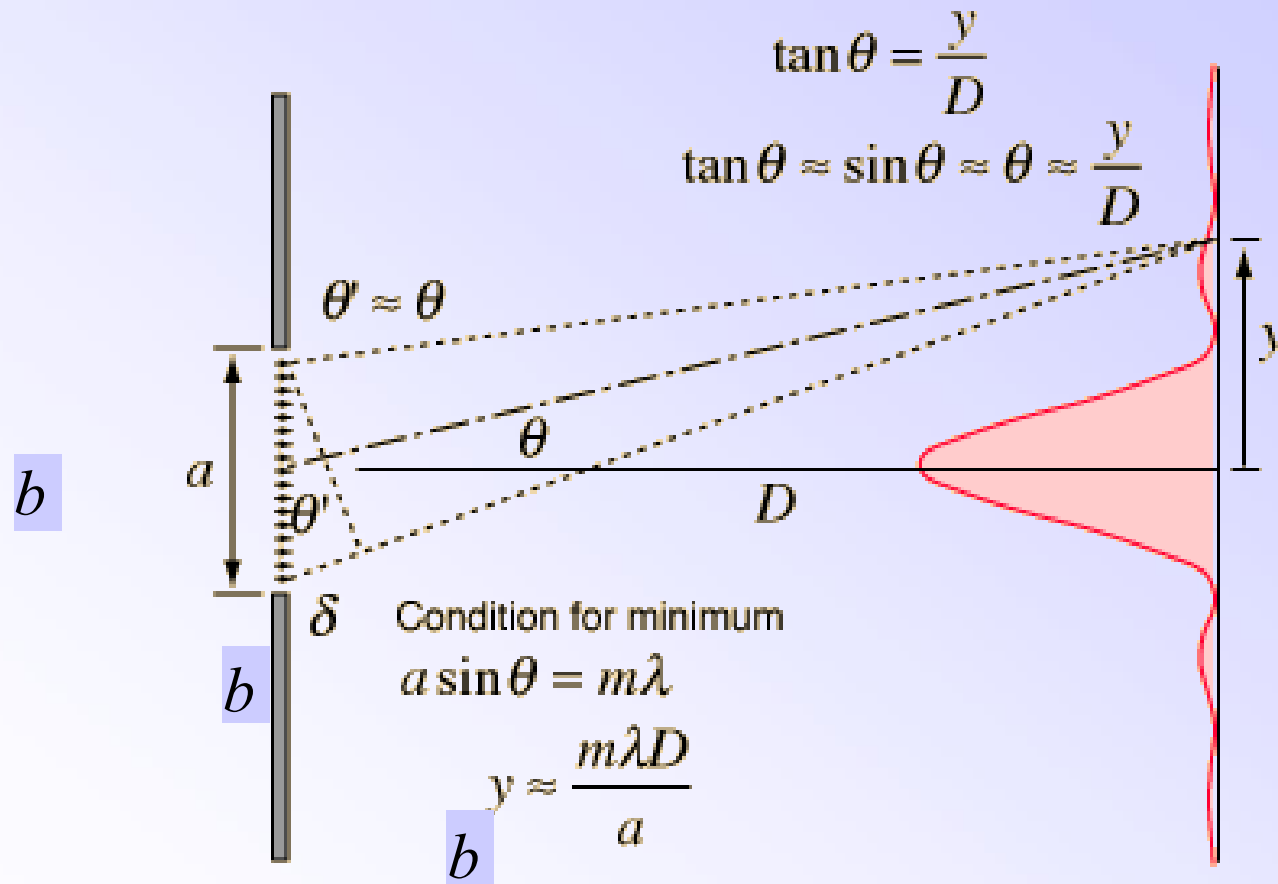
- $\beta = \pm m(\pi/2); \quad m = 1, 2, 3, \dots$

- $\sin \beta = 1 \quad \rightarrow \quad I(\theta) = I(0) \times \dots$

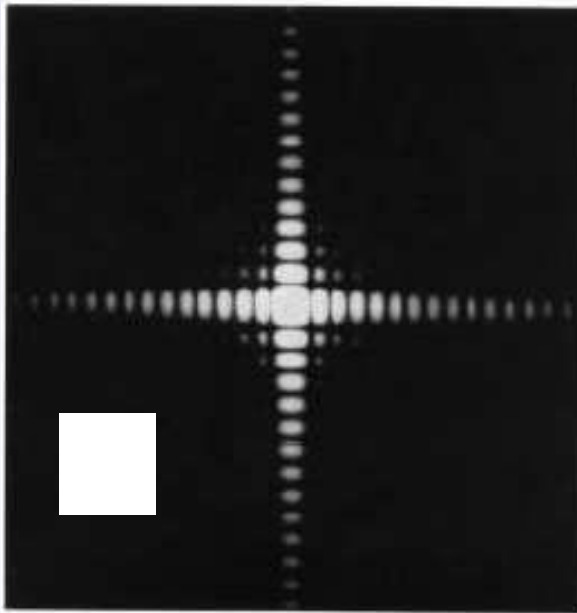
- However for $m = 1$, it does not represent a maximum as it is located between the central maximum and the first order maxima.

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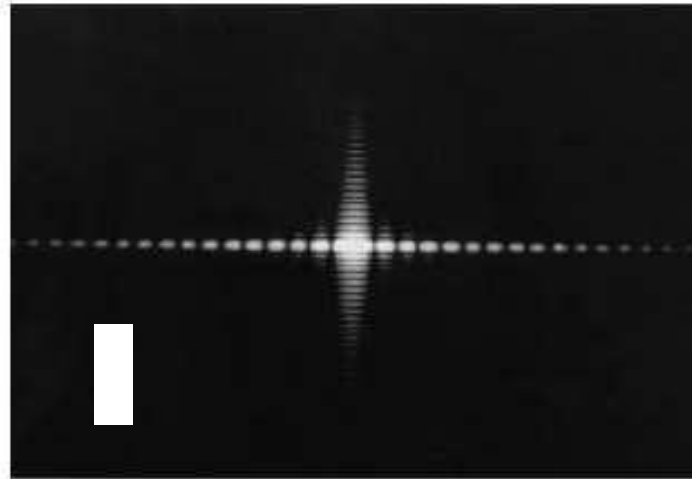




Rectangular Aperture



Diffraction image of a point source (square aperture)



Diffraction image of a point source (rectangular aperture)

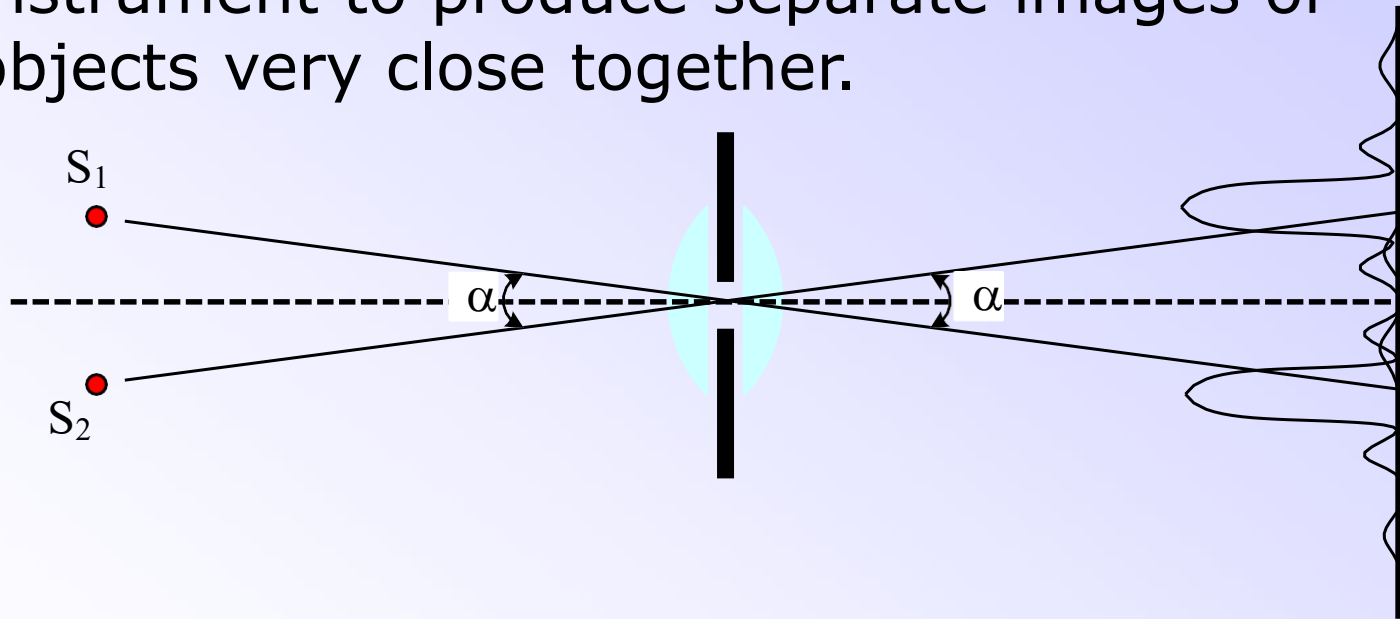
Square aperture (left)

Rectangular Aperture (right)

- The intensity of the pattern is concentrated principally in two directions coinciding with the sides of the aperture.
- There exist an inverse relationship between the slit width and the scale of the pattern.

Resolving Power with a Rectangular Aperture

- Resolving power – the ability of an optical instrument to produce separate images of objects very close together.



- Two sources S_1 and S_2 form images S_1' and S_2' .
- The images are single-slit diffraction pattern.

- The angular separation α of the central maxima is equal to the angular separation of the sources.
- The images are well resolved when each central maxima falls exactly on the *2nd minimum* of the adjacent pattern.
 - This is the smallest possible value of α .
 - If α is above this minimum value, the images are well resolved.
 - If α is less than its minimum value, the images are not resolved.
- For this just resolved situation

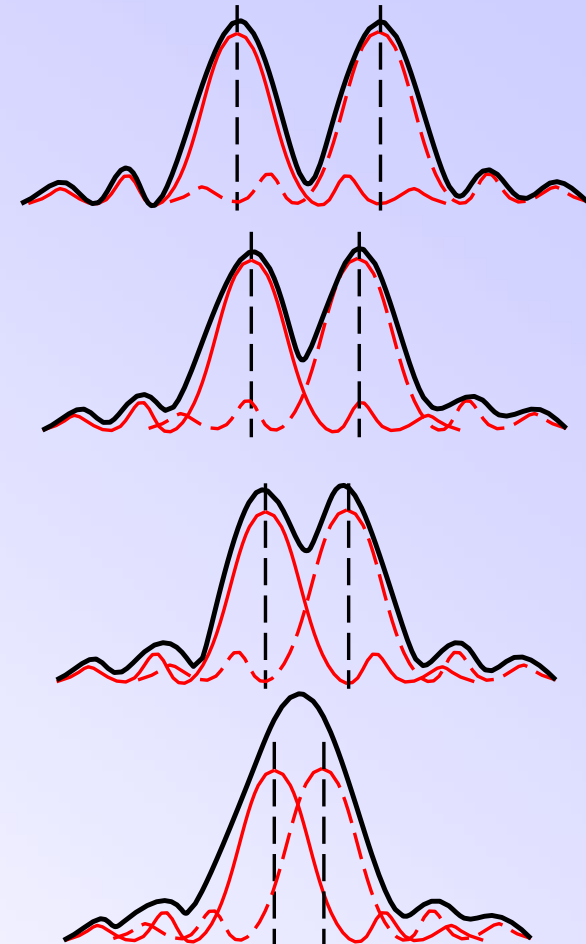
$$\beta = 2\pi$$

or

$$b \sin \theta = (2)\lambda$$

$$\theta = \frac{2\lambda}{b}$$

- The angular separation α of the central maxima is equal to the angular separation of the sources.
- The images are *well resolved* when each central maxima falls exactly on the 2^{nd} minimum of the adjacent pattern.
 - This is the smallest possible value of α which will give zero intensity between the strong maxima in the resultant pattern.
 - As α is less than this value, the images move closer, the intensity between the maxima will rise, until no minima remains in the centre.
 - The images are well resolved in the 1st two figure; the 3rd one is *just resolved* and the last one is *not resolved*.



- For the just resolved situation, the central maxima of one pattern falls exactly on the 1st minimum of the 2nd pattern (slide 25)

$$\beta = \pi$$

or

$$b \sin \theta = \lambda$$

$$\theta_1 = \frac{\lambda}{b}$$

- It is noted that for this situation, the curves cross at $\beta = \pi/2$ for either pattern. As such

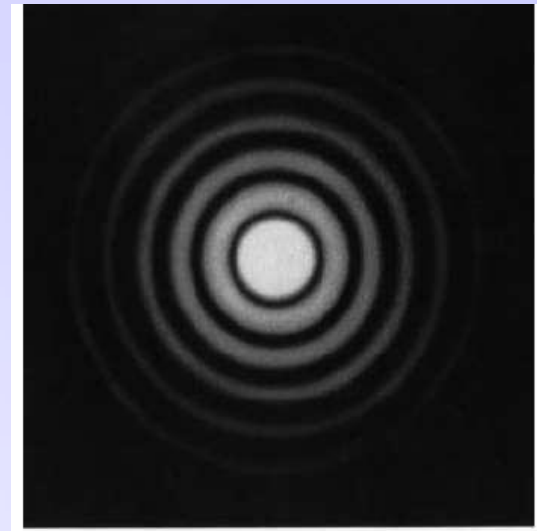
$$\left(\frac{\sin \beta}{\beta} \right)^2 = \frac{4}{\pi^2} = 0.4053$$

- The intensity at the centre of the resultant minimum for diffraction fringes separated by θ_1

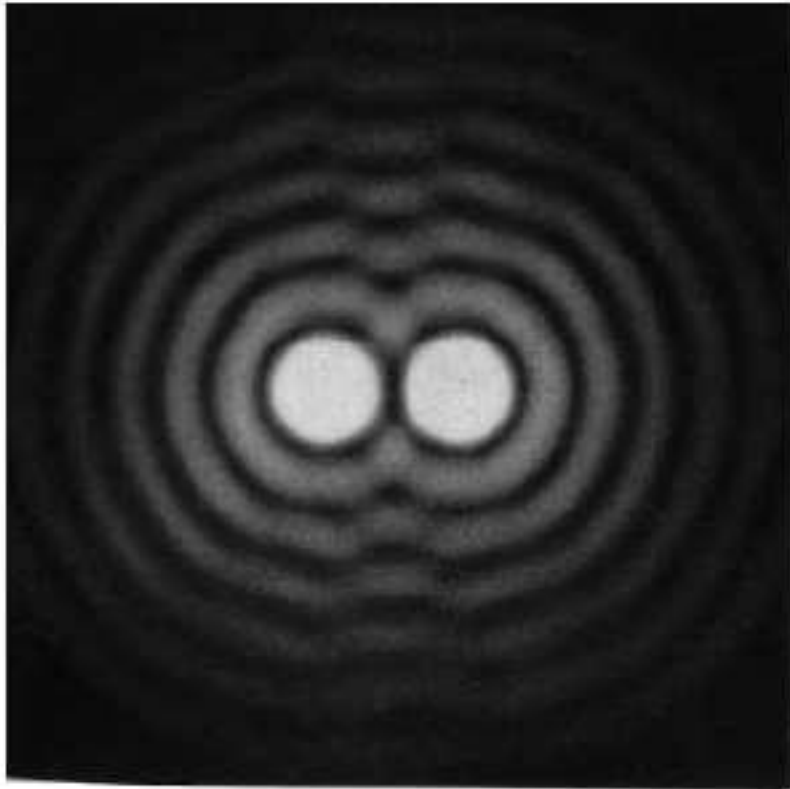
- The intensity of one of the patterns at the centre of the resultant minimum for diffraction fringes separated by θ_1 is 0.4053, thus, the resultant intensity is 0.8106 or $4/5$ of maximum value.
- The difference by this amount is easily detected.
- However, the minimum value increases rapidly as the separation between the maxima decreases.
- It was then decided that $\alpha = \theta_1$ be set as the criterion for the resolution of two diffraction pattern, better known as *Rayleigh's criterion*.

Circular Aperture

- The diagram shows the diffraction pattern formed by plane waves going pass a circular aperture.
- It consist of a bright ring (Airy's disk) surrounded by a number of fainter rings.
 - The rings shade gradually fall off at the edges, being separated by circles of zero intensity.
 - The intensity distribution is very much the same as that which would be obtained with the single-slit pattern, rotated about in the centre of an axis perpendicular to the central maximum.



Rayleigh Resolution Limit



Separated images of 2 Incoherent point sources

Two stars well separated (left)

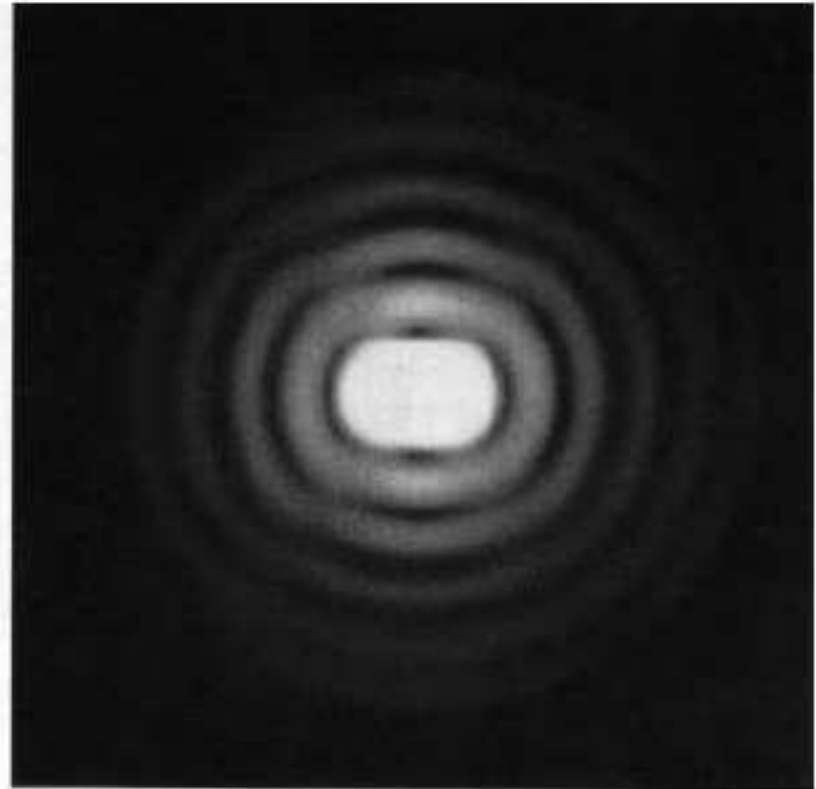


Image of a pair of incoherent point sources at the limit of resolution

Two stars at Rayleigh limit (right)

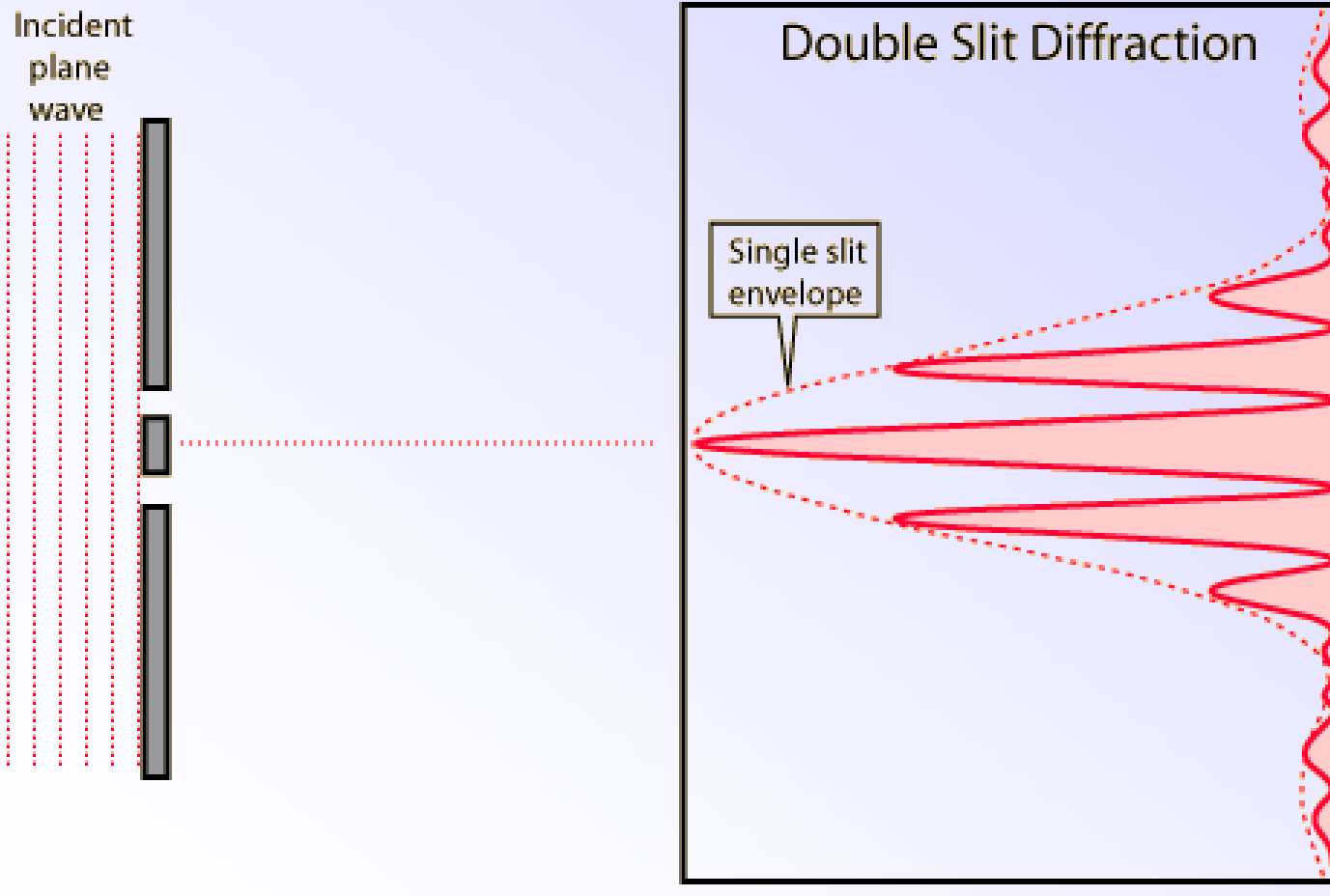
- It was shown that for a rectangular aperture, the angular separation is given by $m\lambda/b$, where m is a whole number starting from unity.
- A similar formula can be stated for the circular aperture but in this case m are not integers (Jenkins & White, page 329, table 15B).
- Extending Rayleigh's criterion for the resolution of diffraction patterns to the circular aperture, the patterns are said to be resolved when the central maximum of one falls on the dark ring of the other.
- The minimum angle of resolution is therefore

$$\theta_1 = 1.220 \frac{\lambda}{D}$$

where D is the aperture diameter.

Double Slits

- The double-slit setup is the same as that of the single slit, but replace the opaque screen with one that has two slits.



- The two slits of width, b , have a centre-to-centre separation of a .
- Each aperture by itself would produce the same single-slit diffraction on the viewing screen.
 - These two wave are coherent.
 - The secondary wavelets will be coherent as well.
- The contribution from the two slits would overlap, interference occurs.
 - The result is then a rapidly-varying double slit interference fringe modulated by a single-slit diffraction pattern.



- The total contribution to the electric field at an arbitrary point P on the viewing screen, according to Fraunhofer approximation, can be shown as

$$E = 2bC \left(\frac{\sin \beta}{\beta} \right) \cos \alpha \sin(\omega t - kR + \alpha)$$

where b – width of slit; $\alpha = \frac{ka}{2} \sin \theta$; and $\beta = \frac{kb}{2} \sin \theta$.

- The irradiance is

$$I(\theta) = I(0) \left(\frac{\sin \beta}{\beta} \right)^2 \cos^2 \alpha$$

$$= 4I_o \left(\frac{\sin \beta}{\beta} \right)^2 \cos^2 \alpha$$

- Interference term
- Diffraction term

- At $\theta = 0$; both $\alpha = \beta = 0$ i.e. $I(0) = 4I_o$.

- If the slits are finite in width but very narrow, the diffraction pattern from either slit will be uniform over a broad central region, and bands resembling the idealized Young's fringes will appear within that region.

- Recall that the diffraction pattern has minima at

$$\beta = \pm\pi; \pm2\pi; \pm3\pi; \dots$$

- That cause the interference pattern to be zero as well.
- Recall that $\beta = \frac{kb}{2} \sin \theta$

the minima occurs when

$$b \sin \theta = m\lambda$$

where $m = 1, 2, 3, \dots$

- The interference term goes to zero when

$$\alpha = \pm \frac{\pi}{2} ; \pm \frac{3\pi}{2} ; \pm \frac{5\pi}{2} ; \dots$$

– As $\alpha = \frac{ka}{2} \sin \theta$, minima occurs when

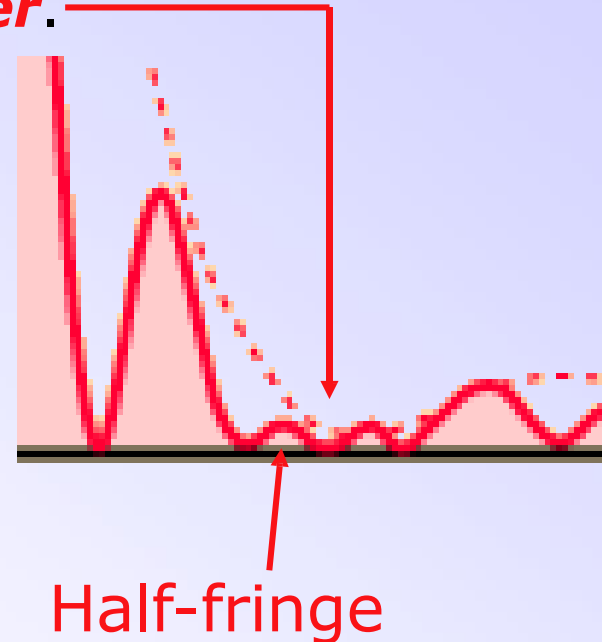
$$a \sin \theta = \left(n + \frac{1}{2}\right) \lambda$$

where $n = 0, 1, 2, \dots$

- The exact positions of the maxima are not given by any simple relation but by their approximate positions neglecting the sine term (only when the slits are very narrow).
- The positions of the maxima are then

$$a \sin \theta = n \lambda$$

- Consider a particular situation where an interference maximum and a diffraction minimum may coincide at the same θ value.
 - In this case, no light is available at that precise position.
 - The interference process is suppressed at that spot.
 - There exist a **missing order**.



Multislits / Diffraction Grating

- Consider the case of N long, parallel, narrow slits, each with width b and centre-to-centre separation a .

- The total optical disturbance at a point on the viewing screen can be shown to take on the form

$$E = bC \left(\frac{\sin \beta}{\beta} \right) \left(\frac{\sin N\alpha}{\sin \alpha} \right) \sin(\omega t - kR + (N-1)\alpha)$$

- The total-flux density distribution function is

$$I(\theta) = I_o \left(\frac{\sin \beta}{\beta} \right)^2 \left(\frac{\sin N\alpha}{\sin \alpha} \right)^2$$

where I_o is the flux density in the $\theta = 0$ direction.

- As with the double-slit case, **principal maxima** occurs when

$$\alpha = 0, \pm\pi, \pm2\pi, \dots$$

or equivalently when

$$a \sin \theta_n = n\lambda; \quad n = 0, \pm 1, \pm 2, \dots$$

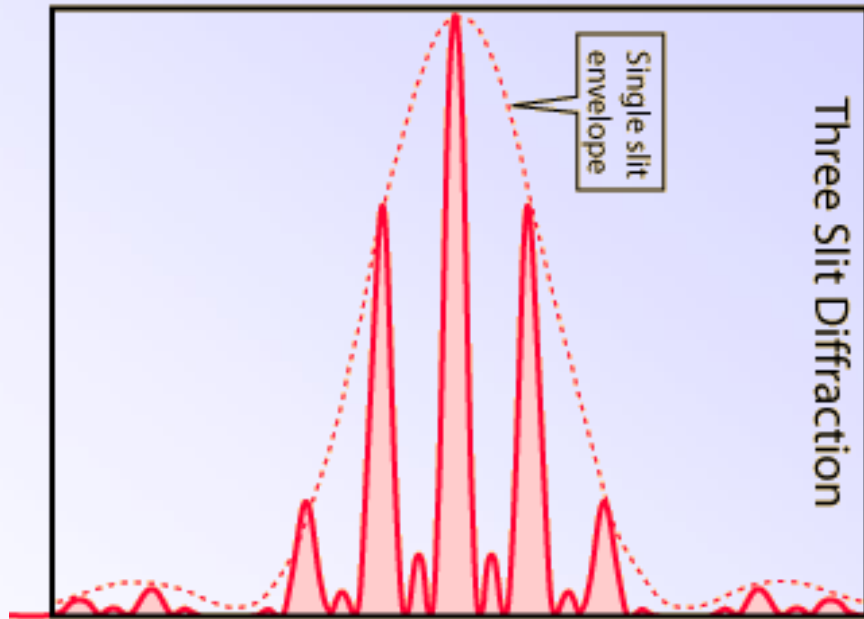
- This is a general expression applicable to all values of N where $N \geq 2$.

- The irradiance has a value of zero when

$$\alpha = \pm \frac{\pi}{N}, \pm \frac{2\pi}{N}, \pm \frac{3\pi}{N} \dots$$

or equivalently when

$$a \sin \theta_n = (n+1) \frac{\lambda}{N}; \quad n = 0, \pm 1, \pm 2, \dots$$



- Between each pair of maxima there will be **secondary maxima**.
 - The number of secondary maxima that exist between a pair of maxima is $N-2$.
 - In some cases secondary maxima cannot be observed by the naked eyes as its irradiance is very low.
 - The intensity of the maxima is the ratio of the square of the number of slits.

