Oscillations

(NEP Semester I - Chapter 4)

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Topics

(1) Solution of Damped SHM in case of

(a) Lightly Damped Motion

(b) Critically Damped Motion &

(c) Heavily Damped Motion with example and application

(2) Derivation of Forced SHM solution and its nature and applications

Objectives

Student will be able to:

- Define the damped motion
- Define the resonance.
- Compare between free, damped and derived oscillations

Damped Oscillations

Where the force is proportional to the speed of the moving object and acts in the direction opposite the motion.

The retarding force can be expressed as:

$R = -bv$ (where b is a constant called damping coefficient)

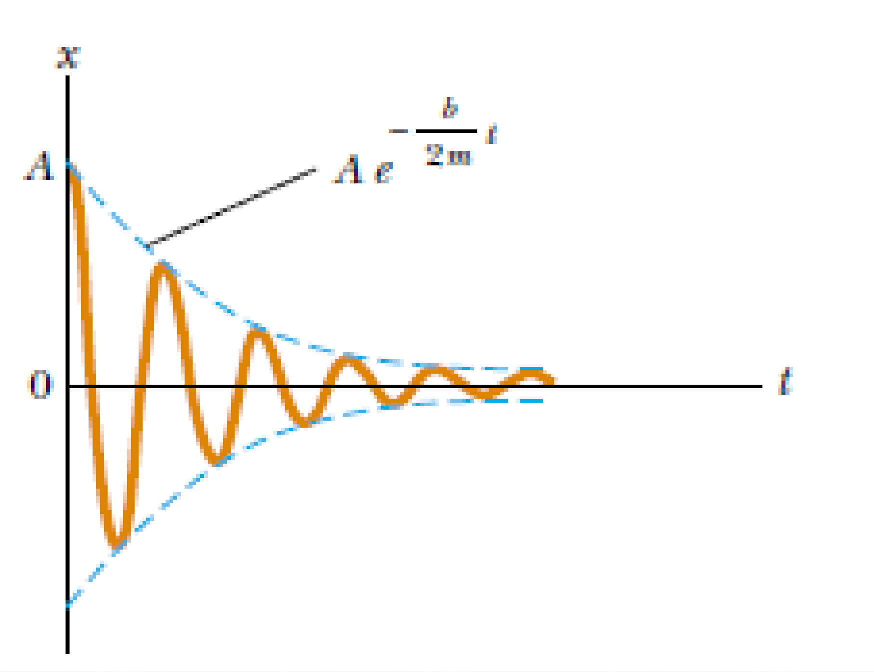
and the restoring force of the system is $-kx$,

then we can write Newton's second law as

$$\sum F_x = -kx - bv_x = ma_x \qquad -kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

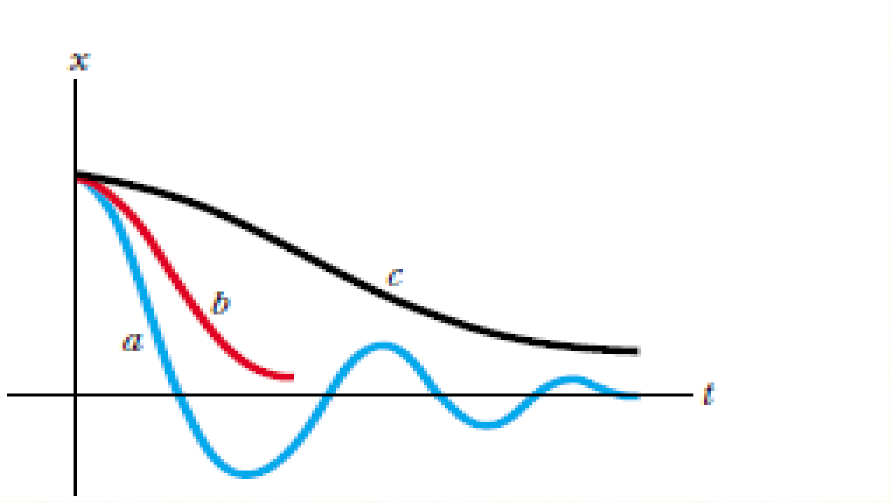
When the retarding force is small compared with the max restoring force that is, b is small the solution is,

$$x(t) = Ae^{-\frac{b}{2m}t}\cos(\omega t + \varphi) \qquad \omega = \sqrt{\frac{k}{m} - (\frac{b}{2m})^2}$$



Represent the position vs time for a damped oscillation with decreasing amplitude with time

The figure shows the position as a function in time of the object oscillation in the presence of a retarding force, the amplitude decreases in time, this system is known as a damped oscillator. The dashed line which defined the envelope of the oscillator curve, represent the exponential factor



The figure. represent position versus time:

•*under damped oscillator*

•*critical damped oscillator*

- Overdamped oscillator.

as the value of "b" increase the amplitude of the oscillations decreases more and more rapidly.

When *b* reaches a critical value b_c ($b_c / 2m = \omega_o$), the system does not oscillate and is said to be critically damped.

And when $b_c / 2m > \omega_o$ the system is overdamped.

Forced Oscillation

For the forced oscillator is a damped oscillator driven by an external force that varies periodically

Where,

$$F(t) = F_o \sin \omega t$$

where ω is the angular frequency of the driving force and F_o is a constant

From the Newton's second law

$$\sum F = ma \rightarrow F_o \sin \omega t - b\frac{dx}{dt} - kx = m\frac{d^2x}{dt^2}$$

$$x = A\cos(\omega t + \varphi)$$

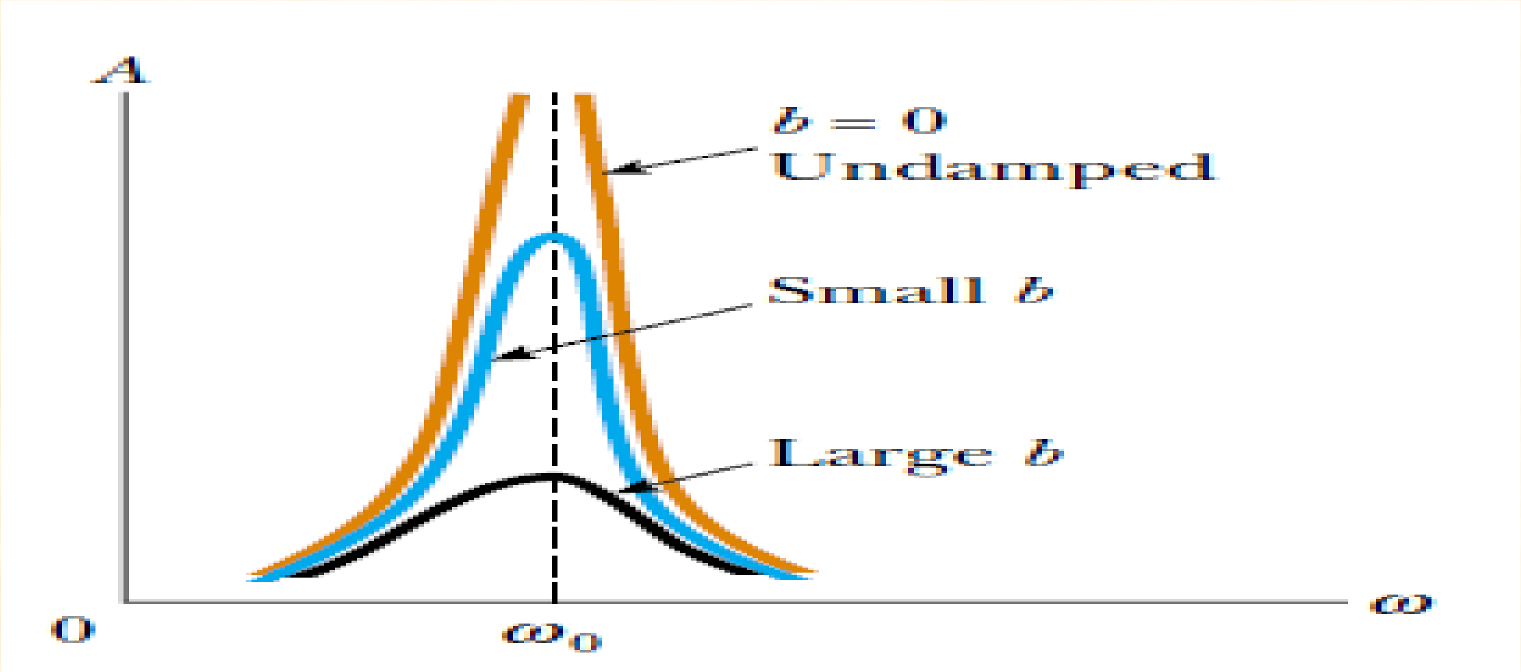
$$A = \frac{F_o / m}{\sqrt{(\omega^2 - \omega_o^2)^2 + \left(\frac{b\omega}{m}\right)^2}}$$

≈

$$\omega_o = \sqrt{k/m}$$ is the natural frequency of the un-damped oscillator (b=0).

The last two equations show the driving force and the amplitude of the oscillator which is constant for a given driving force.

For small damping the amplitude is large when the frequency of the driving force is near the natural frequency of oscillation, or when $\omega \approx \omega_o$ the is called the resonance and the natural frequency is called the resonance frequency.



Amplitude versus the frequency, when the frequency of the driving force equals the natural force of the oscillator, resonance occurs. Note the depends of the curve as the value of the damping coefficient b.

Summary of the chapter

1. The acceleration of the oscillator object is proportional to its position and is in the direction opposite the displacement from equilibrium, the object moves with SHM. The position x varies with time according to,

$$x(t) = A\cos(\omega t + \varphi)$$

2. The time for full cycle oscillation is defined as the period, $T = 2\pi / \omega$

For block spring moves as SHM on the frictionless surface with a period

$$T = 2\pi/\omega = 2\pi\sqrt{\frac{k}{m}}$$

3. The frequency is defined as the number of oscillation per second, is the inverse of the period

$$f = 1/T = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

4. The velocity and the acceleration of SHM as a function of time are

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \varphi) \qquad a = \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \varphi)$$

We not that the max speed is Aω , and the max acceleration is Aω^2 .

The speed is zero when the oscillator is at position of x=± A , and is a max when the oscillator is at the equilibrium position at the equilibrium position *x=0*.

5. The kinetic energy and potential energy for simple harmonic oscillator are given by,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \varphi) \qquad U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega t + \varphi)$$

The total energy of the SHM is constant of the motion and is given by

$$E = \frac{1}{2}kA^2$$

6. A simple pendulum of length L moves in SHM for small angular displacement from the vertical, its period is $T = 2\pi\sqrt{L/g}$

7. For the damping force *R = - bv*, its position for small damping is described by

$$x(t) = Ae^{-\frac{b}{2m}t}\cos(\omega t + \varphi) \qquad \omega = \sqrt{\frac{k}{m} - (\frac{b}{2m})^2}$$

8. If an oscillator is driving with a force $F(t) = F_o \sin \omega t$

it exhibits resonance, in which the amplitude is largest when driving frequency matches the natural frequency of the oscillator.

What is the effect on the period of a pendulum of doubling its length?

$$T_L = 2\pi\sqrt{L/g}$$

$$L \to 2L$$

$$\therefore T_{2L} = 2\pi\sqrt{2L/g}$$

$$\therefore T_{2L} = \sqrt{2}T_L = 1.414T_L$$