Gravitation

(NEP Semester I - Chapter 3)

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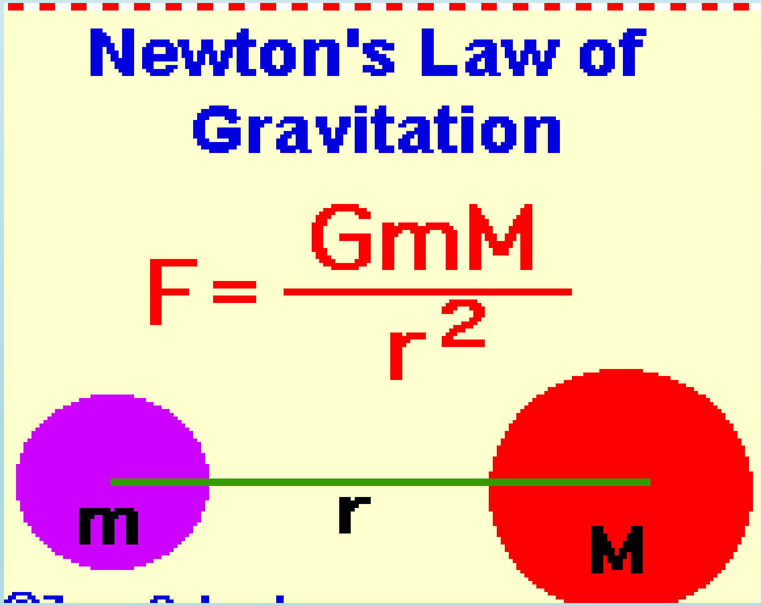
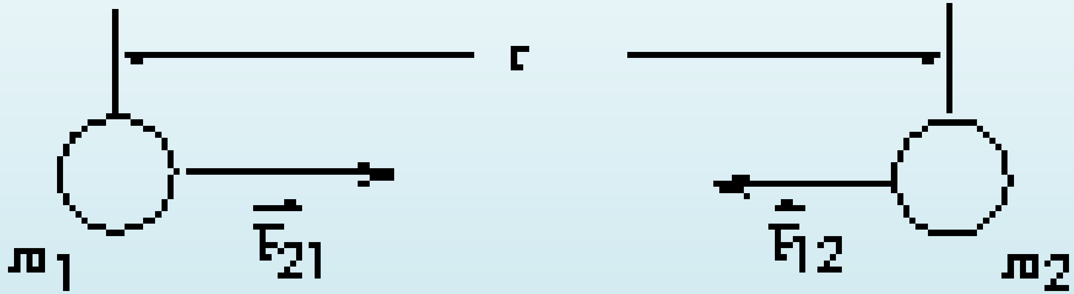
1. Newton's Law of Gravitation

Every particle of matter attracts every other particle with a force which is directly proportional to the product of the masses and inversely proportional to the square of the distance between them.



1. Newton's Law of Gravitation (contd.)



$$F = \frac{G m_1 m_2}{r^2}$$

G is called the Universal Gravitational Constant

G = 6.67 x 10^{-11} N-m^2/kg^2

G is constant throughout the Universe and G does not depend on the medium between the

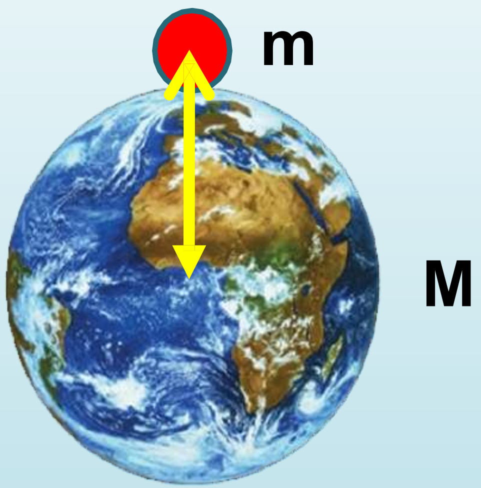
2. Difference between G and g

G	g
G is the Universal Gravitational Constant	g is acceleration due to gravity
$G = 6.67 \times 10^{-11}$ N m^2/kg^2	Approx value $g = 9.8$ m / s^2. Value of g varies from one place to another on the Earth.
Constant throughout the Universe	Changes every place on a planet. E.g., on the Moon, the value of g is $1/6^{th}$ of that on the Earth's surface.

3. Relation between G and g

- Let M = mass of the Earth
- m = mass of an object on the surface of the Earth
- g = acceleration due to gravity on the Earth's surface
- R = radius of the Earth

3. Relation between G and g (contd.)



Weight of the object is the gravitational force acting on it.

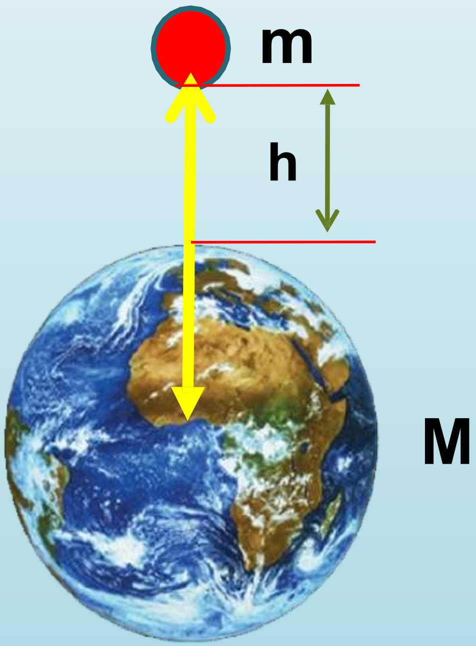
$$weight\ of\ the\ object = gravitational\ force$$

$$mg = \frac{GMm}{R^2}$$

$$g = \frac{GM}{R^2}$$

..............................(1)

3. Relation between G and g (contd.)



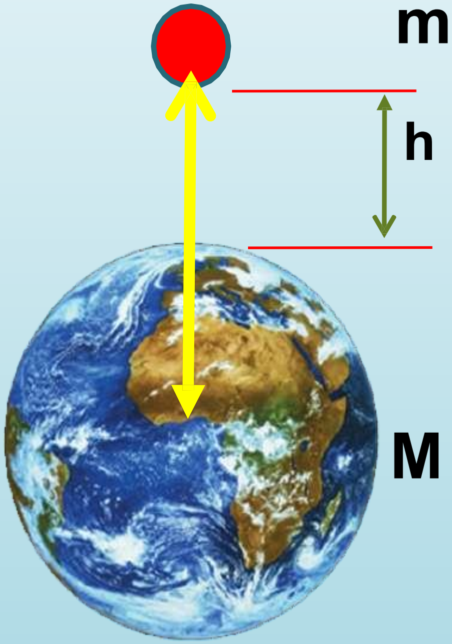
At height h from the surface of the Earth's surface, acceleration due to gravity is g_h

At height h,
Weight of object = gravitational force

$$\cancel{m} g_h = \frac{GM\cancel{m}}{(R+h)^2}$$

$$g_h = \frac{GM}{(R+h)^2} \quad \text{.................................(2)}$$

3. Relation between G and g (contd.)



Dividing (2) by (1) we get,

$$g_h = g\left(\frac{R}{R+h}\right)^2$$

Thus, 'g' is independent of the mass of the object.

4. Kepler’s Laws of Motion

Johannes Kepler

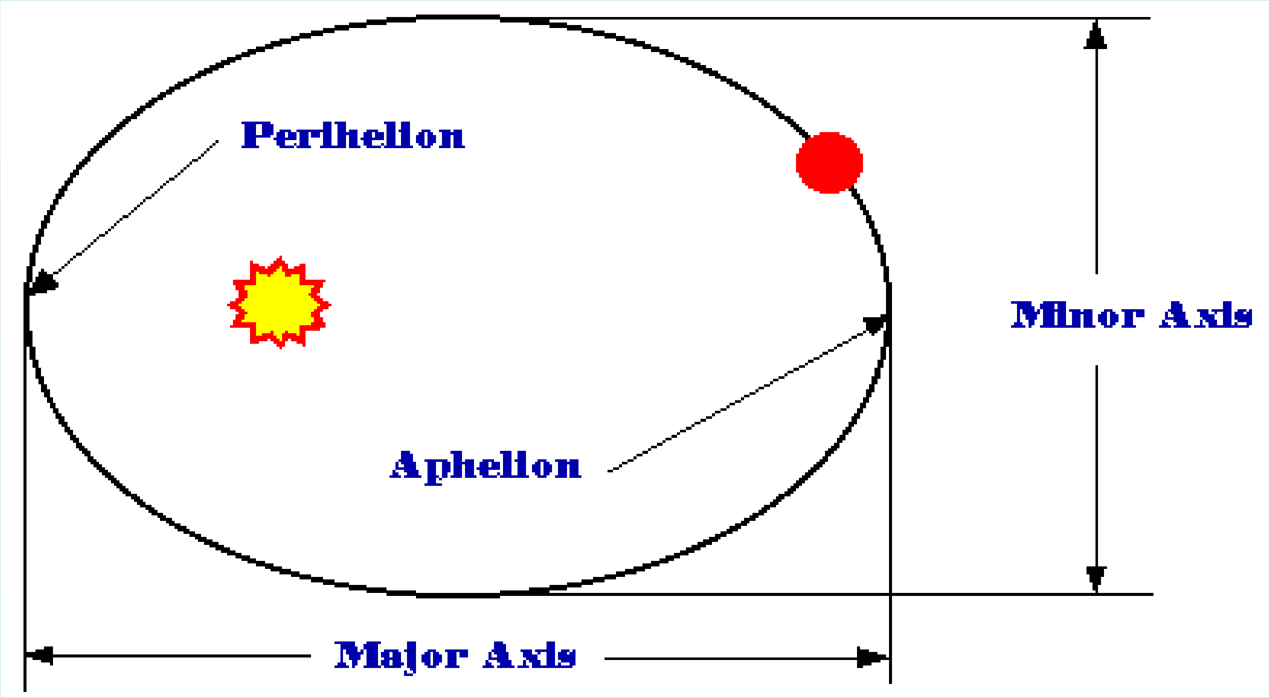
Born:
December 27, 1571

Died:
November 15, 1630

German
Mathematician,
Astronomer
Astrologer.

4.1. Kepler's First Law – Law of Orbit

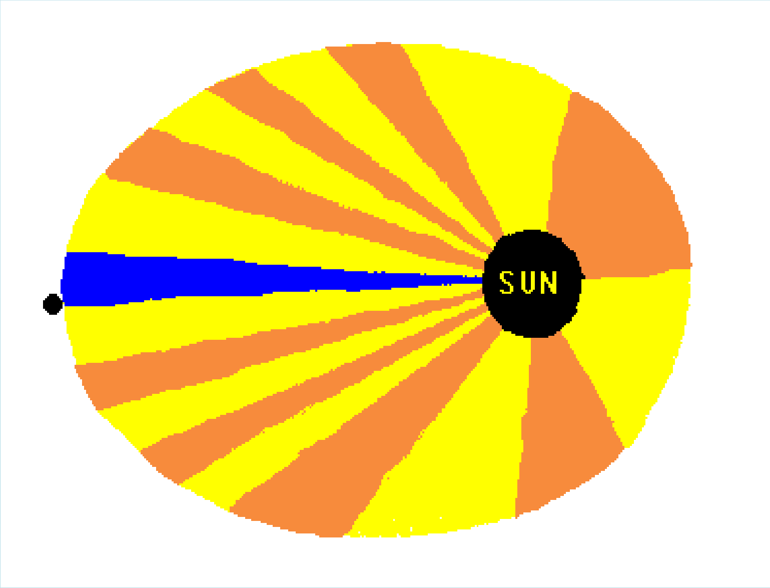
Every planet revolves in an elliptical orbit around the Sun, with the Sun situated at one focus of the ellipse.



4.2. Kepler's Second Law - Law of Equal Areas

The radius vector drawn from the Sun to any planet sweeps out equal areas in equal intervals of time. This law is called the law of areas.

The areal velocity of the radius vector is constant.



4.3. Kepler's Third Law - Law of Period

The square of period of revolution of the planet around the Sun is directly proportional to the cube of the semi-major axis of the elliptical orbit.

$$T^2 \propto r^3$$

According to this law, when the planet is closest to the Sun, its speed is maximum and when it is farthest from the Sun, its speed is minimum.

5. Projection of a Satellite

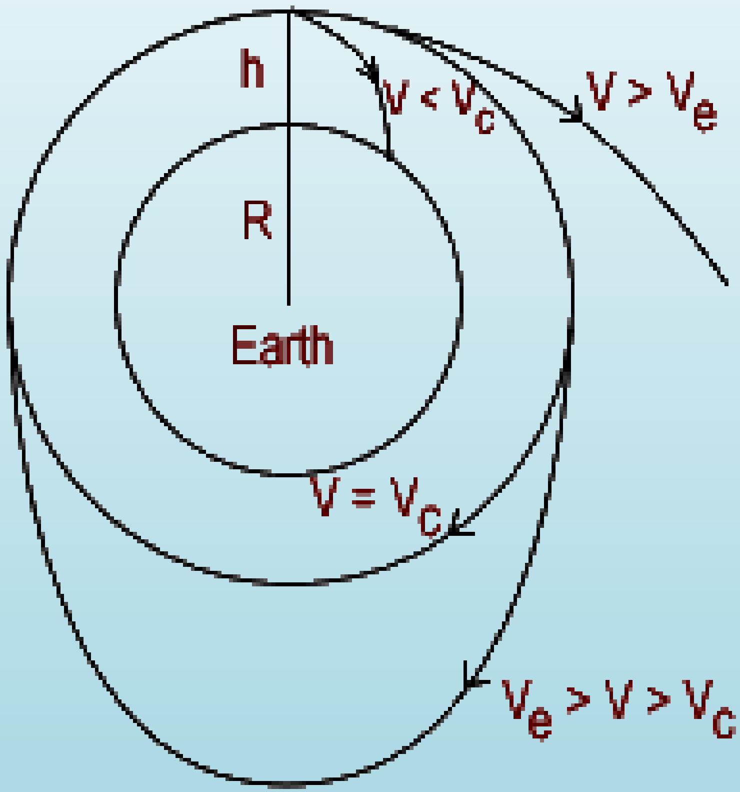
Why is it necessary to have at least a two stage rocket to launch a satellite?

A rocket with at least two stages is required to launch a satellite because

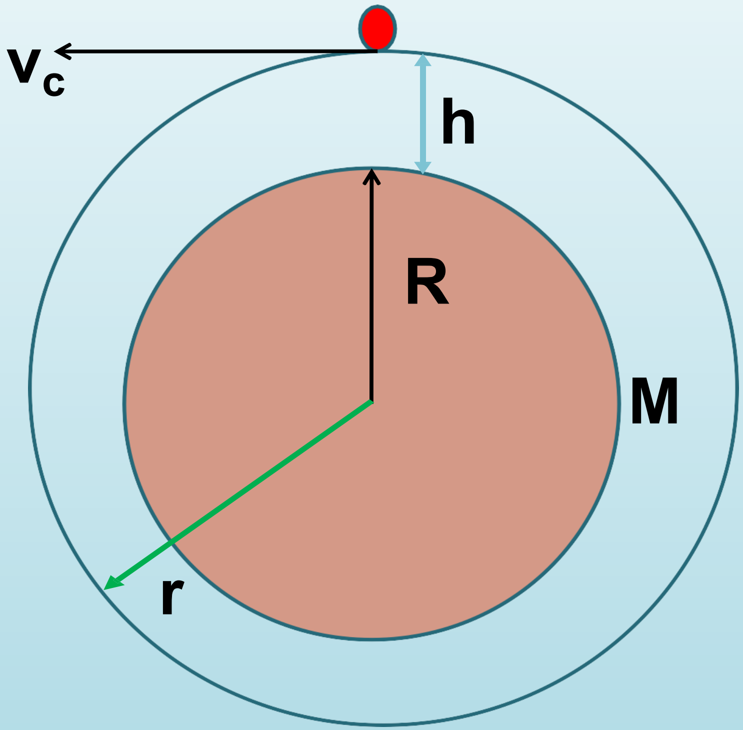
- The first stage is used to carry the satellite up to the desired height.
- In the second stage, rocket is turned horizontally (through 90 degrees) and the satellite is fired with the proper horizontal velocity to perform circular motion around the earth.

6. Critical Velocity of a Satellite

The horizontal velocity with which a satellite should be projected from a point above the earth's surface, so that it orbits in a circular path around the earth is called the orbital velocity or critical velocity (V_c) of the satellite.



6. Critical Velocity of a Satellite (contd.)



M = mass of the Earth

R = radius of Earth

m = mass of satellite

h = height of the satellite above Earth's surface

r = R + h, where r is the distance of the satellite from the center of the Earth

V_c = critical velocity

The centripetal force necessary for the circular motion of the satellite around the Earth is provided by the gravitational force of attraction between the Earth and the satellite.

6. Critical Velocity of a Satellite (contd.)

Centripetal force = Gravitational force

$$\frac{mv_c^2}{r} = \frac{GMm}{r^2}$$

$$v_c^2 = \frac{GMm}{r}$$

$$v_c = \sqrt{\frac{GM}{r}} \qquad \text{..........................(1)}$$

6. Critical Velocity of a Satellite (contd.)

Factors on which Critical Velocity of a satellite depends:

1. Mass of the planet
2. Radius of the planet
3. Height of the satellite

Critical velocity is not dependent on the mass of the satellite as **m** does not appear in the above equation

6. Critical Velocity of a Satellite (contd.)

But we know that

$$g_h = \frac{GM}{(R+h)^2}$$

$$GM = g_h (R+h)^2$$

Substituting this value in eqn (1), we get,

$$v_c = \sqrt{\frac{g_h (R+h)^2}{(R+h)}}$$

$$v_c = \sqrt{g_h (R+h)} \quad \text{..(2)}$$

6. Critical Velocity of a Satellite (contd.)

Assignment 1:

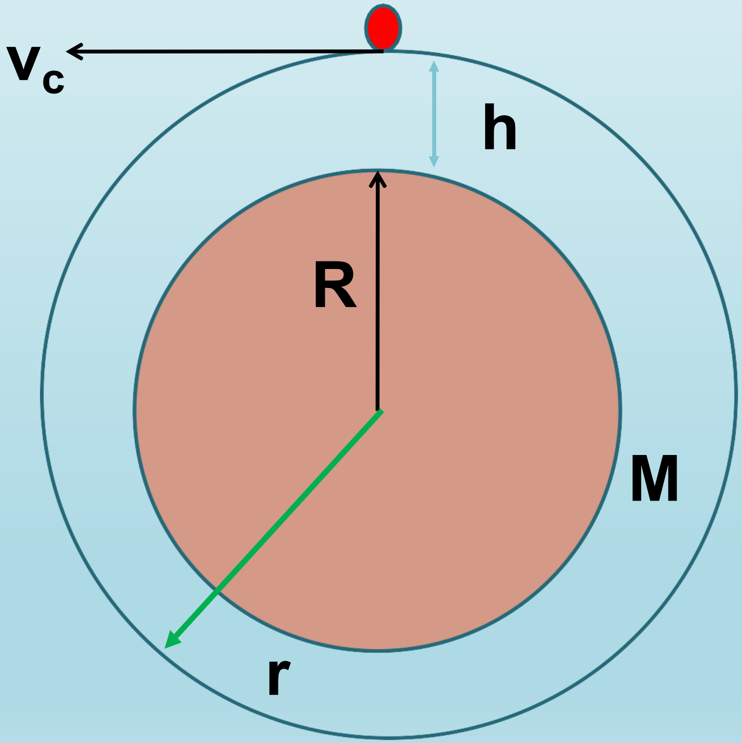
Modify eqn (2) to find the critical velocity of a satellite orbiting very close to the surface of the Earth ($h << R$)

Assignment 2:

How does the critical velocity (or orbital velocity) of a satellite vary with an increase in the height of the satellite above the Earth's surface?

7.Time Period of a Satellite

The time taken by a satellite to complete one revolution around the earth is called its periodic time or time period.



M = Mass of the Earth
R = Radius of Earth
m = Mass of satellite
h = Height of the satellite above Earth's surface
r = R + h, where r is the distance of the satellite from the center of the Earth
V_c = Critical velocity

7. Time Period of a Satellite (contd.)

Distance covered by the satellite in 1 revolution = Circumference of the circle

Time taken to cover this distance is the time period.

So, Critical speed $V_c = \frac{circumference}{periodic\ time}$

or, $V_c = \frac{2\pi r}{T}$

But $v_c = \sqrt{\frac{GM}{r}}$

7. Time Period of a Satellite (contd.)

$$\sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$$

Squaring both sides, we get

$$\frac{GM}{r} = 4\frac{\pi^2 r^2}{T^2}$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

As $\frac{4\pi^2}{GM}$ is a constant,

so we get,

$$T^2 \propto r^3$$

Thus, the square of the period of revolution is directly proportional to the cube of the radius of its orbit.

7.Time Period of a Satellite (contd.)

Factors on which Time Period of a satellite depends:

1. Mass of the planet
2. Radius of the planet, and
3. Height of the satellite from the planet's surface

Period of the satellite does not depend on the mass of the satellite.

Assignment:

(1) Obtain an expression for the time period of a satellite in terms of g_h.

(2) For a satellite close to the earth, calculate the period of revolution in minutes.